



MORTech 2019

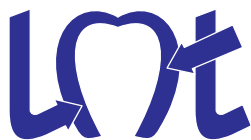
5th international Workshop Reduced Basis, POD and PGD Model Reduction Techniques

November 20-22, 2019 @ FIAP Jean Monnet Conference Center
PARIS

An IACM Special Interest Conference



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co-organized by
LMT (ENS Paris-Saclay/CNRS/Université Paris-Saclay)
PIMM (Arts et Métiers ParisTech, CNRS, CNAM)



Reduced Basis, POD and PGD Model Reduction Techniques

Co-Chairmen

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D. Néron - LMT (ENS Paris-Saclay/CNRS/Université Paris-Saclay)

F. Chinesta - PIMM (Arts et Métiers ParisTech/CNRS/le cnam)

Scope

Mechanics, like other domains, continues to supply numerous engineering problems which, despite the impressive progress of computational simulation techniques, remain intractable today. RB, PGD and other model reduction methods are leading to a new generation of high-performance computational tools which provide solutions to engineering problems which are inaccessible to standard codes based on classical and well-established numerical techniques.

The workshop is intended to be a meeting ground for the various contributors, including mechanicians, applied mathematicians and other researchers and engineers involved in testing and computation. The Workshop should provide answers to such questions as:

- What are the maturity and the benefits of RB and POD/PGD methods?
- What are also their limitations?
- What engineering challenges, especially in mechanics, could be addressed in the near future?
- What are the key scientific issues?

Main topics

- Convergence, verification and adaptive approaches
- ROM for large numbers of parameters and nonlinear problems
- Uncertainty quantification and propagation
- Multiscale and multiphysics problems
- Quasi-real-time simulations: control, optimization, design...
- Data-based and data-driven ROM
- Non-invasive approaches
- Engineering applications

Local organizing and scientific committee

P.-A. Boucard (ENS Paris-Saclay), L. Chamoin (ENS Paris-Saclay), F. Chinesta (ENSAM Paris), C. Farhat (Stanford Univ.), P.-A. Guidault (ENS Paris-Saclay), P. Ladevèze (ENS Paris-Saclay), D. Lange (NAFEMS), Y. Maday (Sorbonne Université), D. Néron (ENS Paris-Saclay).

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Practical information



The Lisbonne and Berlin rooms are located on level -2.
Coffee breaks will be held at level 0.
The "Wine and Cheese" Poster session will take place on level 0.
Finally, lunches will be taken at level 1 where the restaurant is located.



For your presentation, a MacBook Pro (Catalina OS) with Acrobat Reader DC, Office for Mac (PowerPoint) 2019 and Keynote 9 is available.

VLC media player 3 for videos is also installed.

Please bring your presentation material with USB memory device and install it on the computer **before the beginning of the session.**

You can use your own computer as soon as you have ensured that it is working properly on the beamer.



Oral presentations will be **30 min long**, including **5 minutes** of discussion.

Session Chairs will enforce these times strictly and will stop presentations that run over time.



A wifi access is available in each conference room.

The SSID (name) of the WiFi network is: **WIFIAP**

It is an open wireless network so you don't need any password.



You need assistance?

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Day 1 – Wednesday, November 20

Morning sessions

07:45-08:20 **Registration**

08:20-08:30 **Opening**

Session 1 Room Lisbonne Chair: Pierre Villon

08:30-09:00 **Charbel Farhat**
Enablement of Nonlinear Multiscale Modeling: In-Situ Adaptive vs. Coupon Test Analogy Trainings and Reduced-Order Bases vs. Neural Networks

09:00-09:30 **Peter Benner**
Realization-independent Reduced-Order Modeling of Dynamical Systems

09:30-10:00 **Albert Cohen**
Reduced modeling for manifold sensing

10:00-10:30 **Pierre Ladevèze**
Model order reduction for nonlinear problems involving complex time-varying loadings

10:30-11:00 **Coffee break**

Session 2 Room Lisbonne Chair: Andrea Manzoni

11:00-11:30 **Francisco Chinesta**
Advances in multi-scale, multi-domain and data-based PGD

11:30-12:00 **David Ryckelynck**
Computer vision for reduced-order modeling of macroscopic mechanical tests

12:00-12:30 **Anthony Nouy**
Approximation and learning with tree tensor networks

12:30-13:00 **Wim Desmet**
Model order reduction techniques to enable digital twins of high-dynamic mechatronic systems

13:00-14:15 **Lunch**

Day 1 – Wednesday, November 20

Afternoon sessions

Session 3 Room Lisbonne Chair: Kathrin Smetana

- 14:15-14:45 **Samuele Rubino**
Numerical stabilization for the violation of the LBB condition in POD-ROM
- 14:45-15:15 **Abdallah El Hamidi**
A complete proof of the convergence of alternating minimization in PGD methods
- 15:15-15:45 **Antonio Falco**
Topological Methods in Model Reduction

Session 4 Room Berlin Chair: Ludovic Chamoin

- 14:15-14:45 **Frédéric Legoll**
Certified computations with PGD model reduction for multiscale problems
- 14:45-15:15 **Kenan Kergrene**
A posteriori error estimation and adaptivity for a goal-oriented PGD method
- 15:15-15:45 **Chady Ghnatios**
Non-intrusive model order reduction technique for real time simulation of industrial applications
- 15:45-16:15 **Coffee break**

Session 5 Room Lisbonne Chair: Anthony Gravouil

- 16:15-16:45 **Piotr Breitkopf**
Incremental POD and Custom Integration Schemes for Hyper-reduced nonlinear dynamics
- 16:45-17:15 **Pierre-Alain Boucard**
Multi-fidelity metamodeling using reduced order models

Session 6 Room Berlin Chair: Antonio Falco

- 16:15-16:45 **Ludovic Chamoin**
Real-time stochastic data assimilation using PGD: application to damageable structures
- 16:45-17:15 **Olga Mula**
Reconstruction of blood flows with Doppler ultrasound images
- 17:15-17:30 **Break**

Poster session Room Lisbonne Chair: Virginie Ehlacher, David Néron

- 17:30-18:15 **Virginie Ehlacher, David Néron**
Presentation of posters
- 18:15-20:30 Wine and Cheese evening and discussions around posters

Day 2 – Thursday, November 21

Morning sessions

Session 7 Room Lisbonne Chair: Adnan Ibrahimbegovic

- 08:30-09:00 **Hermann Matthies**
Analysis of Stochastic Parameterised Reduced Order Methods
- 09:00-09:30 **David Néron**
Parameter-multiscale PGD for high dimensional parametric spaces
- 09:30-10:00 **Antonio Huerta**
Generalized solutions (PGD-based Computational Vademecums) for parametric studies in industrial CFD
- 10:00-10:30 **Yvon Maday**
Data assimilation with PBDW approach for real time prediction of evolution phenomenon
- 10:30-11:00 **Coffee break**

Session 8 Room Lisbonne Chair: Florian De Vuyst

- 11:00-11:30 **Adnan Ibrahimbegovic**
Scale coarsening model reduction for failure mechanics of concrete composites: meso-scale to stochastic macro-scale transition
- 11:30-12:00 **Mathilde Chevreuil**
Learning in tree-based tensor formats for uncertainty quantification
- 12:00-12:30 **Kathrin Smetana**
Randomized Model Order Reduction
- 12:30-13:00 **Damiano Lombardi**
An adaptive hierarchical local HOSVD method

Session 9 Room Berlin Chair: Pierre-Alain Boucard

- 11:00-11:30 **Jean-Louis Duval**
From Non-Intrusive model order reduction to advanced real-time engineering
- 11:30-12:00 **Michel Rochette**
Industrial and Medical Digital Twins powered by Reduced Order Modelling
- 12:00-12:30 **Jean Ragusa**
Application of Proper Generalized Decomposition to Neutron Governing Laws
- 12:30-13:00 **Jean-François Ganghoffer**
Symmetry analysis and equivalence transformations in constitutive modeling
- 13:00-14:15 **Lunch**

Day 2 – Thursday, November 21

Afternoon sessions

Session 10	Room Lisbonne	Chair: Frédéric Legoll
14:15-14:45	Annika Robens-Radermacher	Coupling PGD model reduction with importance sampling using adaptive subset simulation for reliability analysis
14:45-15:15	Anthony Gravouil	Isogeometric analysis suitable trivariate models generation dedicated to reduced order modeling with geometric parameters
15:15-15:45	Florian De Vuyst	Physics-guided data-driven reduced-order modeling for nonlinear dynamical problems

Session 11	Room Berlin	Chair: Olga Mula
14:15-14:45	Virginie Ehlacher	Model reduction in Wasserstein spaces for transport problems
14:45-15:15	Olivier Zahm	Reducing the input parameter dimension using gradient information
15:15-15:45	Tommaso Taddei	A registration method for model order reduction: data compression and geometry reduction
15:45-16:15	Coffee break	

Session 12	Room Lisbonne	Chair: Mathilde Chevreuril
16:15-16:45	Ruth V. Sabariego	Reduced-order models of nonlinear magneto-quasi-static problems. Alternatives to DEIM-POD?
16:45-17:15	Enrique Delgado	Reduced Basis Method for the Boussinesq VMS-Smagorinsky model

Session 13	Room Berlin	Chair: Jan Hesthaven
16:15-16:45	Macarena Gómez Mármó	On the computation of Proper Generalized Decomposition modes of parametric elliptic problems
16:45-17:15	Isabel Sánchez Muñoz	Numerical analysis on the computation of modes for the Proper Generalized Decomposition to parametric elliptic problems

Banquet		
18:30-20:30	Visit of Musée d'Orsay	
20:30-22:30	Dinner at Musée d'Orsay	

Day 3 – Friday, November 22

Morning sessions

Session 14 Room Lisbonne Chair: Julien Yvonnet

- 08:30-09:00 **Wing Kam Liu**
Mechanistic Machine Learning Methods for Mechanical Science and Design/Optimization of Lightweight Material Systems
- 09:00-09:30 **Elias Cueto**
Learning fluid mechanics from data
- 09:30-10:00 **Gianluigi Rozza**
Perspectives in Reduced Order Methods in Computational Fluid Dynamics: the effort of increasing the Reynolds number
- 10:00-10:30 **Frédéric Feyel**
Physical ROMs: how to accommodate nonparametrized variability, nonintrusivity, performance and error indication for large scale industrial applications?
- 10:30-11:00 **Coffee break**

Session 15 Room Lisbonne Chair: Piotr Breitkopf

- 11:00-11:30 **Julien Yvonnet**
A two-scale FE2 method using neural networks
- 11:30-12:00 **Maria Cinefra**
Development of reduced structural theories for composite plates and shells via machine learning

Session 16 Room Berlin Chair: El Hamidi Abdallah

- 11:00-11:30 **Andrea Manzoni**
Nonlinear dimensionality reduction of parametrized PDEs by reduced basis methods and deep learning techniques
- 11:30-12:00 **Jan Hesthaven**
Nonintrusive reduced basis models through machine learning

Discussion Room Lisbonne Chair: David Néron

- 12:00-12:45 **Discussion, conclusion**
- 12:45-14:00 **Lunch - End**

POSTERS
Day 2 - Thursday, November 21
Evening session

Entezami Alireza

Low-order feature extraction technique and unsupervised learning for SHM under high-dimensional data

Clara Argerich Martin

Application of Code2Vect-S-PGD in an industrial framework: Acoustic prototypes

Julia Brunken

Model order reduction for a kinetic Fokker-Planck equation

Cristina Caravaca García

Reduced Basis Method applied to the Smagorinsky Model

Fabiola Cavaliere

A Proper Generalized Decomposition approach with Inertia Relief (PGD-IR) for the static analysis of geometrically parametrized unconstrained structures

Ana Gabriela Chavez Castillo

Characterization of insulating materials by implementation of a reduced order model in the inverse procedure

Sridhar Chellappa

Adaptive parameter sampling for the Reduced Basis method

Philippe De Brabander

On model reduction applied to a Trefftz method to simulate the transient dynamic response resulting from a shock

Khalil El Rai

Distortion of a part during Selective Laser Melting using the Proper Generalized Decomposition

Amélie Fau

Model order reduction strategy for fatigue damage analysis

Benjamin Gaume

Modal reduction adapted to heat transfer with radiation in a furnace

Paul-William Gerbaud

New Tools for Data-Driven Computation in Nonlinear Solid Mechanics

Sebastian Grimberg

Hyperreduction of nonlinear Petrov-Galerkin reduced-order models using ECSW

Ruben Ibanez Pinillo

Tensor Model Learner

Frédéric Joly

A modal substructuring method for non-conformal mesh

Steffen Kastian

Proper orthogonal decomposition with an adaptive selection of modes for problems with nonlinearities

Hanane Khatouri

Expected improvement based infill Sampling for full-field multi-fidelity optimization

Ygee Larion

Reduced Basis Approximation and Goal-Oriented Error Estimation for Time-Dependent Coupled Thermo-Hydro-Mechanical System

Florian Mainmear

A spot weld simplified model based on reduced order modeling

Andrew McClellan

Projection-based Model Order Reduction for Model Predictive Control of a Landing Aircraft

Rolando Mosquera

Some geometric and machine learning methods for model order reduction

Christina Nasika

Reduced Model Order Techniques For Data Assimilation In Tailings Dams Monitoring

Hai Nam Nguyen

New numerical tools based on PGD in the context of model identification from full-field measurements

Dionysios Panagiotopoulos

An automated Krylov subspace recycling based model order reduction technique for acoustic BEM systems

Igor Pontes Duff

Automatic generation of minimal and reduced models for structured parametric systems

Jonatha Reis

PGD for 2D elasticity problems, with error bounds and adaptativity

Pieter Reumers

Green's functions of a layered halfspace using the proper generalized decomposition

Iuri Rocha

A domain-based adaptive hyper-reduced framework applied to plasticity and crack growth

Sebastian Rodriguez

A new time multi-scale LATIN-PGD for the treatment of complex seismic excitations

Ward Rottiers

Temporal dimension reduction via parametric state-time formulation

Ronan Scanff

On a non-intrusive version of the LATIN-PGD method for non-linear time-dependent problems

Kelbij Star

FV-based POD-Galerkin reduced order model for boundary control of natural convection flows

Jacinto Ulloa

Multitemporal space-time integration for dissipative solids under cyclic loading

Tatsuro Yashiki

Swirling flow analysis in piping system using proper orthogonal decomposition method and Galerkin Projection

Matteo Zancanaro

Reduced Order Models for incompressible turbulent Navier-Stokes flows in a finite volume environment

ABSTRACTS

Day 1 – Wednesday, November 20

Morning sessions

Enablement of Nonlinear Multiscale Modeling : In-Situ Adaptive vs. Coupon Test Analogy Trainings and Reduced-Order Bases vs. Neural Networks

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Abstract

Heterogeneous materials are often encountered in solid mechanics and structural dynamics problems. The brute force Finite Element (FE) modeling of such problems usually requires extremely fine meshes and therefore is prohibitively expensive. For this reason, alternative approaches such as the multiscale FE² [1] have been developed. Typically, these assume that the material configuration is homogeneous at the macroscale but heterogeneous at the smallest represented scale, and couple the sequence of scales using a pair of localization and homogenization procedures. For many problems however, they remain computationally intensive, if not unaffordable. This is because they require at each but the finest scale the solution of a large number of nonlinear, large-scale, unit cell problems.

To address the issue outlined above, various Projection-based Model Order Reduction (PMOR) frameworks have been developed (for example, see [2] and more recently [3]). In particular, the multiscale PMOR approach presented in [3] was shown to dramatically accelerate the solution of nonlinear dynamic multiscale problems. In this framework, the dimensionality of the governing equations is reduced using the proper orthogonal decomposition method, and computational efficiency is achieved for the evaluation of the nonlinear reduced-order terms using the Energy Conserving Sampling and Weighting (ECSW) hyperreduction method [4, 5, 6]. Training is performed in two steps. First, a microscale hyperreduced model is constructed in-situ in order to achieve significant speedups, even in non-parametric settings. Next, a classical offline-online training approach is performed to build a parametric hyperreduced macroscale model. A notable feature of this approach is the minimization, at the macroscale level, of the training cost using the in-situ hyperreduced microscale model to accelerate snapshot acquisition. A weak

spot of this approach however is the in-situ training itself, which, for sufficiently large macroscale models, requires an adaptive reduction process to keep the global Reduced-Order Basis (ROB) accurate and economical at all times.

This talk will start by presenting such an adaptive process based on the concept of a database of local ROB that is constructed and updated on-the-fly. This process treats the deformation gradient as a vector parameter domain, which enables it to locate each unit cell problem in the database and assign to it online the most appropriate local ROB. Its accuracy is optimized by collecting new snapshots as needed, and updating accordingly the local ROB.

Next, the talk will present a new concept dubbed coupon test analogy training as an alternative to in-situ adaptive training. It will also generalize the PMOR framework originally developed in [3] to include : a treatment of contact featuring the application of a non-negative factorization scheme to the construction of a positive ROB for the contact forces [7]; and the incorporation Neural Network (NN) representations as an alternative to the solution of reduced, FE-based, unit cell problems.

Finally, this talk will demonstrate all machine learning concepts outlined above, whether based on PMOR or NN, for several academic problems as well as for a Mars landing application involving the supersonic inflation of an atmospheric aerodynamic decelerator system that includes a parachute canopy made of a woven fabric.

References

- [1] F. Feyel and J.-L. Chaboche. FE^2 Multiscale Approach for Modelling the Elastoviscoplastic Behaviour of Long Fibre SiC/Ti Composite Material. *Computer Methods in Applied Mechanics and Engineering*, 183(3-4):309-330, 2000.
- [2] J. Yvonnet and Qi-Chang He. The Reduced Model Multiscale Method (R3M) for the Non-linear Homogenization of Hyperelastic Media at Finite Strains. *Journal of Computational Physics*, 223(1):341-268, 2007.
- [3] M. J. Zahr, P. Avery and C. Farhat. A Multilevel Projection-based Model Order Reduction Framework for Nonlinear Dynamic Multiscale Problems in Structural and Solid Mechanics. *International Journal for Numerical Methods in Engineering*, 112(8): 885-881, 2017.
- [4] C. Farhat, P. Avery, T. Chapman and J. Cortial. Dimensional Reduction of Nonlinear Finite Element Dynamic Models with Finite Rotations and Energy-Based Mesh Sampling and Weighting for Computational Efficiency. *International Journal for Numerical Methods in Engineering*, 98(9):625-662, 2014.
- [5] C. Farhat, T. Chapman and P. Avery. Structure-Preserving, Stability, and Accuracy Properties of the Energy-Conserving Sampling and Weighting Method for the Hyper Reduction of Nonlinear Finite Element Dynamic Models. *International Journal for Numerical Methods in Engineering*, 102(5):1077-1110, 2015.
- [6] T. Chapman, P. Avery, P. Collins and C. Farhat, Accelerated Mesh Sampling for the Hyper-reduction of Nonlinear Computational Methods. *International Journal for Numerical Methods in Engineering*, 109(12):1623-1654, 2014.
- [7] M. Balajewicz, D. Amsallem and C. Farhat. Projection-based Model Reduction for Contact Problems. *International Journal for Numerical Methods in Engineering*, 106(8):644-663, 2016.

Realization-independent Reduced-order Modeling of Dynamical Systems

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Abstract

We consider dynamical systems in input-output (I/O) form. The most prominent class of such systems are linear time-invariant systems as the main workhorse in control engineering. Other I/O-systems of particular structure are mechanical (dissipative) systems, delay systems, bilinear systems, polynomial systems, etc. We assume that no realization of the system is available, meaning we have an oracle providing us with the output of the system, given an input. We will discuss novel data-driven methods to compute compact surrogate models of a desired structure using the I/O data only.

References

- [1] P. Benner and P. Goyal. Interpolation-Based Model Order Reduction for Polynomial Parametric Systems. arXiv Preprint arXiv:1904.11891 [math.NA], April 2019.

Reduced modeling for manifold sensing

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Abstract

We consider the problem of reconstructing from m linear measurements the solution u to a partial differential equation where certain parameters are unknown, therefore lying on a solution manifold.

Fast numerical recovery methods have been proposed based on approximation spaces of moderate dimension n which are tailored to the solution manifold, such as reduced bases and POD.

These methods are inherently limited by their linear nature. We propose in this talk several non-linear counterparts.

MODEL ORDER REDUCTION FOR NONLINEAR PROBLEMS INVOLVING COMPLEX TIME-VARYING LOADINGS

Pierre Ladevèze, David Néron, Sebastian Rodriguez and Ronan Scanff

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Simulating the nonlinear mechanical response of structures subjected to complex time-varying loadings (such as seismic loadings or fatigue loadings with a large number of cycles) remains a challenge. Our answer is a new Model Order Reduction method called "the time-multiscale PGD" based on a signal theory involving two times: a macro-time and a micro-one. In addition, this strategy is written in a non-intrusive manner, i.e. it is a first attempt to be compatible with commercial FE softwares and in particular to all the facilities they offer to deal with nonlinearities.

Advances in multi-scale, multi-domain and data-based PGD

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J.L. Duval⁶

¹ ESI Group Chair @ PIMM, ENSAM ParisTech

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⁴ LaCaN, UPC, Barcelona

⁵ I3A, University of Zaragoza

⁶ ESI Group

Abstract

The present work will cover five different topics needing for advanced PGD formulations. The first of them concerns the space separability in domains containing interfaces non-parallel with the coordinate axes [1]. The second concerns a multi-local space separated representation where the continuity along the space is ensured within the partition of unity [2]. The third application concerns time or space multi-scale problems where slow and fast scales are separated, and the solution searched under a separated representation [3]. Again, the partition of unity enables the solution continuity. Then sparse sensing is applied for different purposes [4] and finally data-driven PGD constructors will be considered.

References

- [1] C. Ghnatios, E. Abisset-Chavanne, A. Ammar, E. Cueto, J.L. Duval, F. Chinesta. Advanced spatial separated representations. *Computer Methods in Applied Mechanics and Engineering*.
- [2] R. Ibanez, E. Abisset-Chavanne, F. Chinesta, A. Huerta, E. Cueto. A local, multiple Proper Generalized Decomposition based on the Partition of Unity. *International Journal for Numerical Methods in Engineering*.
- [3] R. Ibáñez, A. Ammar, E. Cueto, A. Huerta, J.L. Duval, F. Chinesta Multi Scale Proper Generalized Decomposition based on the Partition of Unity. *International Journal for Numerical Methods in Engineering*.
- [4] R. Ibanez, E. Abisset-Chavanne, E. Cueto, A. Ammar, J.L. Duval, F. Chinesta. Some applications of compressed sensing in computational mechanics. Model order reduction, manifold learning, data-driven applications and nonlinear dimensionality reduction. *Computational Mechanics*.

Computer vision for reduced-order modeling of macroscopic mechanical tests

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Abstract

In this work, computer vision enables recommending a reduced order model for fast stress prediction according to various possible mechanical environments. This approach is applied on a macroscopic part by using a digital image of a mechanical test. We propose a hybrid approach that simultaneously exploits machine learning and a physics-based model, in mechanics of materials [1]. During a machine learning stage, a classification of possible reduced order models is obtained through a clustering of loading environments by using simulation data. The recognition of the suitable reduced order model is performed via a convolutional neural network (CNN) applied to a digital image of the mechanical test. The CNN recommends a convenient mechanical model available in a dictionary of reduced order models. The output of the convolutional neural network being a model, an error estimator is proposed to assess the accuracy of this output. This work opens the route for the model reduction of non parametric problems, where surrogate models are generated on the fly via machine learning techniques.

Such surrogate models are developed for the stress prediction in welding joint having multiple voids. Parametric modeling of all possible configurations of voids, is out of reach. In this case, the loading environment around the voids is extracted from void free simulations, supplemented by tomographic images of the voids. This talk will detail simple algorithmic choices that allowed a realistic mechanical modeling via computer vision.

References

- [1] F. Nguyen, S. M. Barhli, D. Pino Muñoz and D. Ryckelynck, Computer Vision with Error Estimation for Reduced Order Modeling of Macroscopic Mechanical Tests. Complexity, Article ID 3791543, 10 pages, <https://doi.org/10.1155/2018/3791543>, 2018.

Approximation and learning with tree tensor networks

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Abstract

We consider the approximation of high-dimensional functions using model classes of functions in tree based tensor formats [1], in a statistical learning setting. We first recall the definition of these approximation formats, which can be interpreted as neural networks with a particular architecture, and described by a tree tensor networks. Then we present some results on their approximation power for some classes of functions (including standard regularity classes and classes of functions given by compositions of regular functions) and on their statistical complexity. Finally, we present adaptive learning algorithms using these approximation formats [2] and illustrate their performance for supervised or unsupervised learning, and in particular their ability to recover hidden structures of high-dimensional functions.

In collaboration with M. Bachmayr, E. Grelier, B. Michel and R. Schneider.

References

- [1] A. Falcó, W. Hackbusch, and A. Nouy. Tree-based tensor formats. *SeMA Journal*, Oct 2018.
- [2] E. Grelier, A. Nouy, and M. Chevreuil. Learning with tree-based tensor formats. *arXiv e-prints*, page arXiv:1811.04455, November 2018.

Model order reduction techniques to enable digital twins of high-dynamic mechatronic systems

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Abstract

Over the past decades numerical simulation tools have become indispensable in the analysis (the forward problem), and design and monitoring (the inverse problem) of high-dynamics mechanical and mechatronic systems. A particular property in most mechanical systems, is the importance of their dynamic behavior over a broad frequency range, where different ranges are covered by dedicated simulation formalisms. In the lower frequency range, the behavior is often dominated by nonlinear time-dependent behavior, whereas higher-frequency dynamics can be typically described through (linear and nonlinear) frequency domain analysis.

For large scale functional mechanism motion, the low-frequency dynamics where a few distinct modes dominate the response, is typically covered by flexible multibody simulations. Going higher in frequency, often key for e.g. durability, linear and nonlinear time-domain finite-element analyses come to the fore. Beyond this frequency range, (linearized) vibro-acoustic behavior becomes an essential differentiator in mechanical products, which is mostly covered through frequency domain analysis, although in recent years, time-domain auralisation techniques come to the fore as well.

However, as in other disciplines, the computational loads associated with these simulations are often unfeasibly large to make them broadly applicable in engineering practice. From this perspective model order reduction (MOR) techniques can be a key enabler to bring many of these simulation schemes to engineering. In order to comply with the varying model structure and frequency ranges for the different mechanical simulation problems, dedicated model order reduction schemes need to be developed. In this work we present several MOR schemes which allow to tackle specific mechanical problems and highlight the difficulties in applying general purpose MOR schemes in (industrial) mechanical problems.

In the frame of flexible multibody simulation, we present a range of nonlinear local reduced order modeling schemes, namely Global Modal Parameterization and Static Mode Switching, which enable highly efficient, and even real-time, simulation by exploiting the specific large (Lagrangian) motion and small deformation structure of these models. For nonlinear finite element modeling, for example for tire vibration analysis, we present the Multi-Expansion Modal hyper-reduction scheme which exploits an extended modal reduction basis together with an element sampling and weighting scheme. And for the case of high-frequency vibro-acoustics we show how MOR enables effective time-domain simulation such that auralisation and even inverse time-domain identification becomes possible. Finally, as a broad deployment of MOR is often challenging due to the intrusive nature in the original simulation software, we present a Matrix-Free model order reduction scheme which allows accelerated frequency response analysis in a non-intrusive fashion.

Throughout these developments, the aim is not only to limit the final ROM simulation time, but to enable an overall process time reduction, such that engineers can deploy the presented schemes in daily practice to gain more insight in the many complex dynamic phenomena which govern modern mechanical systems and to let the concept of Digital Twins create true added value.

ABSTRACTS

Day 1 – Wednesday, November 20

Afternoon sessions

Numerical stabilization for the violation of the LBB condition in POD-ROM

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Abstract

In this work, we propose a new stabilized projection-based POD-ROM for the numerical simulation of incompressible flows. The new method draws inspiration from successful numerical stabilization techniques used in the context of Finite Element Methods (FEM), such as Local Projection Stabilization (LPS). In particular, the new LPS-ROM is a velocity-pressure ROM that uses pressure modes as well to compute the reduced order pressure, needed for instance in the computation of relevant quantities, such as drag and lift forces on bodies in the flow. The new LPS-ROM circumvents the discrete inf-sup (LBB) condition for the POD velocity-pressure spaces [2], which can be prohibitively expensive in realistic fluid flow applications. Also, the velocity modes does not have to be neither strongly nor weakly divergence-free [3], which allows to use snapshots generated for instance with penalty or projection-based stabilized methods. The numerical analysis of the fully Navier–Stokes discretization for the new LPS-ROM is presented, by mainly deriving the corresponding error estimates [1]. Numerical studies are performed to discuss the accuracy and performance of the new LPS-ROM on a two-dimensional flow past a circular obstacle [4].

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A complete proof of the convergence of alternating minimization in PGD methods

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Abstract

The approximation of solutions to partial differential equations by tensorial separated representations is one of the most efficient numerical treatment of high dimensional problems. The key step of such methods is the computation of an optimal low-rank tensor to enrich the obtained iterative tensorial approximation. In variational problems, this step can be carried out by alternating minimization (AM) techniques, but the convergence of such methods presents a real challenge. There are only two recent published papers dealing with this question: The first result is by A. EL Hamidi et al [1] in which the authors considered a d – dimensional variational linear elliptic equation with a variable diffusion matrix and showed a partial *global* convergence of the alternating minimization scheme. Indeed, the authors made the hypothesis on the uniqueness of the adherence value of the AM – sequence (all convergent AM – subsequences have the same limit). The second result is by T. Chacon et al [2], in which the authors considered a 2 – dimensional variational linear elliptic problem (parameter & space) and showed a partial *local* convergence result of the AM – sequence, under the following two hypotheses : the uniqueness of the adherence value of the AM – sequence and a large enough coerciveness coefficient. In this situation, the authors provide the convergence rate of such sequences.

In the following work, we give a complete proof of the *unconditional* convergence of the alternating minimization scheme of the optimal low-rank tensor computation for d – dimensional variational linear elliptic equations. Also, we provide the "best" choice of the initialization in the alternating minimization process.

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Topological Methods in Model Reduction

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Abstract

The aim of this paper is to discuss about the question of how we can reconstruct a dynamical system for a given time dependent data set. The main goal of *Data Science* is to extract knowledge from large volumes of unstructured data. It uses data management, statistical learning and topological data analysis, among others tools, to derive mathematical models that allows users to take decisions. In particular, certain branches of mathematics are devoted to develop procedures for organizing data sets into understandable structures (even in infinite dimension). One of these mathematical approaches is based in the concept of *shape*. The first historical approach to the mathematical definition of shape is usually considered founded by the Polish topologist Karol Borsuk in his paper [1] about the homotopy properties of compact sets. We recall that the mathematical concept of *homotopy* is related with the problem to decide when two spaces, that is, two sets endowed with a particular geometry (shape), have the same aspect. relates the count of occurrences of patterns within the shape. In this talk we discuss the different approaches to decide when to sets of orbits share the same dynamic type.

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Certified computations with PGD model reduction for multiscale problems

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Abstract

The Multiscale Finite Element Method (MsFEM) is a Finite Element type approach for multiscale problems, where the basis functions used to generate the approximation space are precomputed [1]. They encode fine-scale details of the microstructure, and are thus specifically adapted to the problem at hand. The computation is performed in a two-stage procedure: (i) an offline stage, in which basis functions are computed as solutions to local fine scale problems, and (ii) an online stage, in which the global problem is solved using an inexpensive Galerkin approximation using a coarse mesh.

In this framework, we have recently proposed an a posteriori error estimate using the concept of Constitutive Relation Error [2]. It enables to effectively assess the various error sources (size of the coarse mesh used to discretize the global problem, size of the local fine meshes used to compute the basis functions, . . .), and to drive a robust adaptive algorithm. We have shown how to use this strategy to control the global error [3] or errors on quantities of interest [4].

In the present work, we investigate the additional use of model order reduction within the MsFEM strategy to further decrease the computational costs, in the case of parameterized microstructures. A naive implementation of the MsFEM would indeed lead to re-computing the basis functions for each new value of the parameter, which is prohibitively expensive. To address this issue, we use the Proper Generalized Decomposition (PGD). We show that this approach can be efficiently employed to solve with low computing cost the various local fine-scale problems associated with the computation of the multiscale basis functions. We will illustrate the performance of the coupling between PGD and MsFEM on several examples.

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A posteriori error estimation and adaptivity for a goal-oriented PGD method

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Abstract

The talk will deal with a formulation aiming at adaptively constructing goal-oriented reduced-order models, that is, PGD models built and adapted towards the approximation of quantities of interest. The main idea behind the goal-oriented approach is to formulate a constrained minimization problem that includes refined information in the goal functionals so that the resulting model be capable of delivering enhanced predictions of the quantities of interest [1]. Such a paradigm represents a departure from standard goal-oriented approaches where the model is first derived by minimization of the energy, or of the residual functional, and subsequently adapted via a greedy approach by controlling a posteriori error estimates measured in terms of quantities of interest using dual-based error estimates [2]. In the present approach these dual-based error estimates – or rather the corrections they suggest – are directly incorporated into the constrained primal problem. The formulation is then applied to the PGD method, with the aim of providing cheap and accurate solution maps for quantities of interest of solutions of parametric boundary value problems [3]. Various sources of errors, namely discretization and truncation errors, are controlled using specific error indicators, naturally leading to a greedy adaptive strategy to further optimize the accuracy of the PGD approximation. Numerical examples will illustrate the performance of the proposed methodology.

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Non-intrusive model order reduction technique for real time simulation of industrial applications

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Abstract

This work explores the possibility to use domain decomposition in unseparable geometries or topologies by leveraging smart mappings, thus overcoming known complications in domain decomposition. Often unseparable domains treated with model order reduction techniques, through decomposition domain properties by using SVD compression, may affect the performance or the accuracy of the model reduction technique [1, 2]. Moreover, some parameters cannot be interpolated for real time applications, like for example the point of application of a load on a beam, or the resin injection position in a mold when simulating the Resin Transfer Molding process. The possibility of using a smart mapping or a physically-based morphing of space is leveraged to circumvent these complications and create real-time simulations in unseparable domains and geometries incompatible for interpolation.

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Incremental POD and Custom Integration Schemes for Hyper-reduced nonlinear dynamics

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1. Abstract

Industrial usage of numerical math-based tools such as the finite element method may in some applications become prohibitive due to the computational cost. This is particularly true in the automotive sector when optimizing the shape of a vehicle in crash situations. Model Order Reduction addresses this issue [4]. Most model order reduction methods rely on the construction of a reduced basis to project the model on. The Proper Orthogonal Decomposition (POD) builds a modal basis from solution observations called snapshots. Data are in a first stage taken from full order model runs and then processed in a so-called off-line phase to give the reduced basis which is then used to build the reduced model.

However, some difficulties arise in the POD. In industrial applications, the data generated in the observation phase may become huge and hard to manipulate. Moreover, the computational cost for post-processing this data may as well explode. Another issue concerns the numerical integration schemes, i.e. the position of numerical integration points and the integration weights. Finally, this contribution revisits the selection criterion of the POD modes in the study of transient systems.

2. Incremental SVD

The incremental Singular Value Decomposition originates from streamed visual content [2,3,5]. This method is based on rank one modifications of a given matrix decomposition. In the POD framework it allows post-processing observations as soon as they are available. This method leads to memory savings and possible computational savings during the off-line phase, as data are not kept in memory and redundant or nonrelevant observations are automatically rejected by the method. Another benefit of this method is the ease to manipulate the POD basis by adding new observations or removing others. For the integration of internal forces, we propose a reduced integration scheme, with integration sites and weights obtained as the solution of a Linear Programming problem. This allows us to add to the training set additional constraints such as an exact integration of monomials or of other explicit base functions.

3. Hyper-reduced internal forces computation

Another difficulty encountered when applying POD to explicit simulation of dynamical problems concerns the internal forces computation. In fact, the projection of the internal forces on the reduced basis requires to compute the internal forces in all finite elements. This step is particularly costly when nonlinear material behaviours are taken into account. The hyperreduced schemes [6,7,8] allow to select a representative subset of finite elements, representative enough to compute internal forces. These methods are however based

heuristics. In this presentation we propose an approach based on simplex algorithm with mixed variables: the discrete indicator function and continuous weights, to find optimal quadratures for hyperreduced systems.

4. Modes selection

In opposition to truncating the set of left SVD modes, taken in decreasing order of associated singular values, according to an averaged error indicator, the proposed method takes temporality into account, resulting in a compact, sparse subset of modes. Selection strategies, implemented in the reduced-order version of a legacy nonlinear explicit dynamics Finite Element (FE) code, are compared in terms of displacement fields and energy reconstruction error.

5. Conclusion

The proposed approach has been implemented in ALTAIR solver RADIOSS© [1] and applied to a structural impact problems. Results regarding computation time, developed features such as estimation of the Reduced Basis error of approximation as well as visual representations illustrate this work. The presentation will focus on the integration of such approach in the whole reduction process, highlighting the attractiveness of the method as well as the required developments to make the whole reduction process incremental. Industrial usage of numerical math-based tools such as the finite element method may in some applications become prohibitive due to the computational cost. This is particularly true in the automotive sector when optimizing the shape of a vehicle in crash situations or when simulating the stamping process. Model Order Reduction addresses this issue.

6. References

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Multi-fidelity metamodeling using reduced order models

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Abstract

Engineering simulation provides the best design products by allowing many design options to be quickly explored and tested, but fast-time-to-results requirement remains a critical factor to meet aggressive time-to-market requirements. In this context, using high-fidelity direct resolution solver is not suitable for (virtual) charts generation for engineering design and optimization.

Metamodels are commonly considered to explore design options without computing every possibilities, but if the behavior is nonlinear, a large amount of data is still required. A possibility is to use further data sources to generate a multi-fidelity surrogate model [1]. Low-fidelity models are cheaper because they are usually a simplification of high-fidelity models. This simplification can be done in different ways, for example linearizing the system, using averaged results in one dimension, using simpler physics models or geometry, less refined domains or partially converged results. Here, model reduction is used to generate low- and high-fidelity responses. Model reduction techniques constitute one of the tools to bypass the limited calculation budget by seeking a solution to a problem on a reduced order basis (ROB). The purpose of the present work is an online method for generating a multi-fidelity metamodel nourished by calculating the quantity of interest from the basis generated on-the-fly with the LATIN-PGD framework [2, 3] for elasto-viscoplastic problems [4].

Low-fidelity fields are obtained by stopping the solver before convergence, and high-fidelity information is obtained with converged solution. In addition, the solver ability to reuse information from previously calculated PGD basis is exploited [5].

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Real-time stochastic data assimilation using PGD: application to damageable structures

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Abstract

The work is placed into the framework of data assimilation in structural mechanics. It aims at developing new numerical tools in order to permit real-time and robust data assimilation that could then be used in various engineering activities. A specific targeted activity is the implementation of DDDAS (Dynamic Data Driven Application System) applications in which a continuous exchange between simulation tools and experimental measurements is envisioned to the end of creating retroactive control loops on mechanical systems. In this context, and in order to take various uncertainty sources (modeling error, measurement noise,..) into account, a powerful and general stochastic methodology with Bayesian inference is considered [1]. However, a well-known drawback of such an approach is the computational complexity which makes real-time simulations and sequential assimilation some difficult tasks.

The research work thus proposes to couple Bayesian inference with attractive and advanced numerical techniques so that real-time and sequential assimilation can be envisioned. First, PGD model reduction [2] is introduced to facilitate the computation of the likelihood function, the uncertainty propagation through complex models, and the sampling of the posterior density. PGD builds a multi-parametric solution in an offline phase and leads to cost effective evaluation of the numerical model depending on parameters in the online inversion phase [3]. Second, Transport Map sampling [4] is investigated as a substitute to classical MCMC procedures for posterior sampling. It is shown that this technique leads to deterministic computations, with clear convergence criteria, and that it is particularly suited to sequential data assimilation. Here again, the use of PGD model reduction highly facilitates the process by recovering gradient and Hessian information in a straightforward manner [5]. Third, and to increase robustness, on-the-fly correction of model bias is addressed in a stochastic context using data-based enrichment terms.

The overall cost-effective methodology is applied and illustrated on a specific test-case dealing with real-time model updating for the control of a mechanical test involving damageable concrete structures with full-field measurements.

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Reconstruction of blood flows with Doppler ultrasound images

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Abstract

Over the past years, several fast reconstruction algorithms based on reduced models have been proposed to address the state estimation problem of approximating an unknown function u of a Hilbert space V from measurement observations. Most strategies are however based on linear mappings where the reduced model is built a priori and independently of the observation space and the measurements. In this work we explore some nonlinear extensions that take these elements into account in the construction of the basis. The methodology is applied to the reconstruction of 3D blood flows from Doppler ultrasound images. The example not only shows the good performance of the nonlinear methods, but it also illustrates the potential of the methodology in medicine as tool to process and interpret data in a systematic manner which could help to build more robust and individualized diagnostics.

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ABSTRACTS

Day 2 – Thursday, November 21

Morning sessions

Analysis of Stochastic Parameterised Reduced Order Methods

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ABSTRACT

Many systems involve models which depend on parameters. These parameters may either be uncertain quantities, leading to uncertainty quantification (UQ), or they may be design parameters to be used in some kind of design optimisation. In any case, the computational model may have to be evaluated many times for different parameters in the computational process. This leads to the desire to replace models which are costly to evaluate – so-called high-fidelity models (HFM, also called simulator) – with computationally cheaper ones, which give comparable results with maybe some little error. These are typically reduced order models (ROMs), also called surrogate-, meta-, or proxy models, or emulators.

Normally, the ROM would be either required to represent the “most important” features of the HFM, or to preserve some of the original physics, and this would include different aspects like the governing equation, a Hamiltonian structure, dynamical stability, etc.

The stochastic view is different in some respects. One postulates a ROM of a certain form (which may embody physical principles), but is mainly taken to allow ease of computation or analysis, e.g. a linear model – and then one has to choose some parameters of the ROM in such a way that the output of the ROM and HFM match in a certain way. This is at this level not too different of what was previously mentioned for physics based models. But now the choice of parameters is performed in a stochastic way, i.e. using some kind of Bayesian approach. This may or may not be constrained by physical considerations.

Parameter-multiscale PGD for high dimensional parametric spaces

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Abstract

Reduced-order models are powerful engineering tools that allow to reduce drastically computational time for complex problems and build “virtual charts” by solving parametrized problems once for all. However, their calculation remains problematic for problems involving many parameters and classical algorithms are limited to about twenty parameters. A new algorithm, in solid mechanics, was proposed in [1]: the so-called “parameter-multiscale PGD”. This method is based on the classical PGD, a model reduction technique using separated variable representations to approximate high dimensional spaces, but the physics of the problem is used to build a more structured representation. Saint-Venant’s Principle, which highlights two different levels of parametric influence, allows to introduce a multiscale description of the parameters to separate a macro- and a micro-scale. First introduced in [1] using a Weak-Trefftz Discontinuous Method in space, [2] presents an adaptation of the algorithm compatible with classical finite element solvers. In this presentation, the procedure is described and 3D numerical examples with up to one thousand parameters are discussed.

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Generalized solutions (PGD-based Computational Vademecums) for parametric studies in industrial CFD

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ABSTRACT

The computational cost of parametric studies currently represents the major limitation to the application of simulation-based engineering techniques in a daily industrial environment. This work presents the first nonintrusive implementation of the proper generalized decomposition (PGD) in OpenFOAM, for the approximation of parametrized incompressible Navier-Stokes equations [1]. The key feature of this approach is the seamless integration of a reduced order model (ROM) in the framework of an industrially validated computational fluid dynamics software.

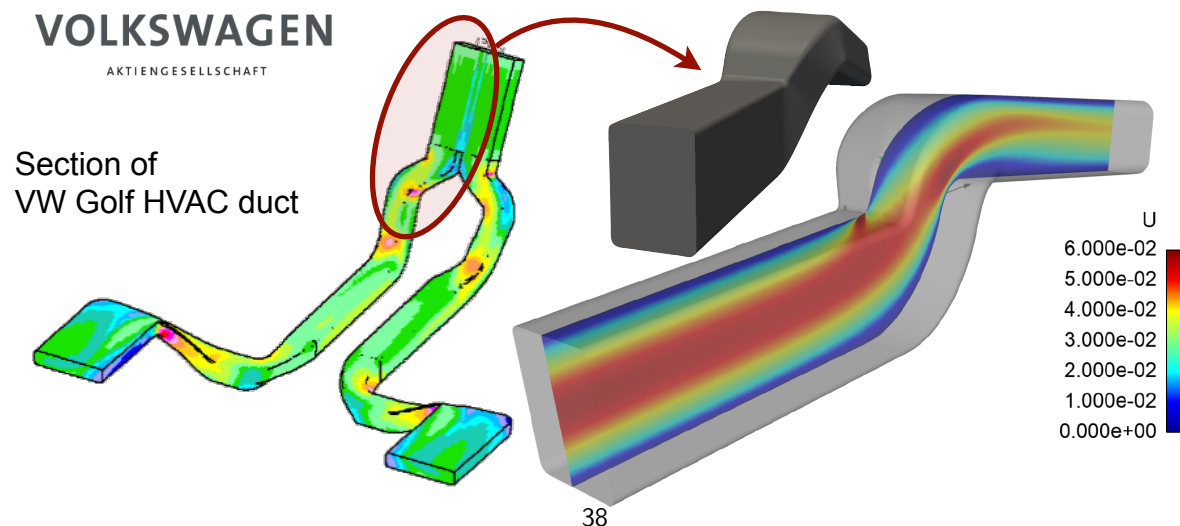
This is of special importance in an industrial environment because in the online phase of the PGD ROM [2] the description of the flow for a specific set of parameters is obtained simply via interpolation of the generalized solution, without the need of any extra solution step. This is one of the major advantages of the PGD-based computational vademecums.

On the one hand, the spatial problems arising from the PGD separation of the unknowns are treated using the classical solution strategies of OpenFOAM, namely the semi-implicit method for pressure linked equations (SIMPLE) algorithm. On the other hand, the parametric iteration is solved via a collocation approach.

The resulting ROM is applied to several benchmark tests of laminar incompressible Navier-Stokes flows, in two and three dimensions, with different parameters affecting the flow features. Eventually, the capability of the proposed strategy to treat industrial problems is verified by applying the methodology to a parametrized flow control in a realistic geometry of interest for the automotive industry.

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Data assimilation with PBDW approach for real time prediction of evolution phenomenon

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Abstract

Reduced basis techniques allow rapid simulations of parameter dependant PDE's using ad'hoc basis functions acquired either from compression methods like SVD or POD, or thanks to greedy approaches (see, eg [3], [1]).

These discretization methods provide accurate solutions to the model, but the precision, of course, depends on the quality of the model that most of the times suffers from bias, due to phenomenon that have been neglected and not taken into account. This bias can be revealed from measures on the phenomenon during a validation process.

The PBDW (parameterized-background data-weak) method allows to improve the reconstruction of the state by using both the data and the model. The model serves as a guideline for the reconstruction and the data allow for corrections. This variational assimilation method was introduced in [2] for steady problems.

In this talk we shall present some new results in this frame dealing with time-dependent problems, the time providing a new direction of assimilation. In particular an approach for proposing extrapolation in time will be presented with some different applications related to improvement of qualitatively good models to provide quantitatively good ones.

This work is done in collaboration with Ludovic CHAMOINS, Germain GONG and Willy HAIK

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Scale coarsening model reduction for failure mechanics of concrete composites: meso-scale to stochastic macro-scale transition

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Abstract

In this paper we deal with constructing best possible scale bridging when passing the detailed information on failure mechanics provided by the meso-scale plasticity model towards the corresponding reduced model at macro-scale. This is accomplished by using Bayesian inference resulting with the probability distribution of reduced model parameters represented as random fields. Such a technique, normally used to find an optimal scheme for data compression in a dynamic analysis of sequence of data, is here recast as the optimal scheme for scale bridging in order to provide reduced model that is capable of providing the best representation for the quantity of interest. The procedure is first developed for concrete meso-scale model (see [1]), for the case when response is not only governed by elasticity, but also by plasticity with hardening typical of fracture process zone and by plasticity with softening typical of localized failure mode. We show that the optimal macro-scale model is not necessarily the same for each phase of inelastic response. Yet, we also show how to combine them all together at the level of a particular finite element with enhanced predictive properties.

We discuss in detail the model reduction for a macro-scale concrete model that will have the local (point-wise) plasticity criterion for fracture process zone with parameters defined as random variables, along with the embedded-discontinuity finite element localized failure criterion with parameters also defined as random variables. The reinforced bars or short fibers can also be included within an enriched version of the proposed framework for localized failure (see [2]).

We consider two sources of uncertainty in development of this scale bridging: i)

uncertainties in the choice of reduced model macro-scale parameters and ii) uncertainties in the microstructure of meso-scale model. For former source of uncertainty, the proposed approach applied to fixed microstructure but with different loading programs. In elastic regime, this merely confirms the standard homogenization-type results for material properties governing elastic response. The real value of the proposed approach is revealed in handling plastic hardening and localized softening response phase, given the most adequate choices for the distance function measuring the scale separation. For latter source of uncertainty with different realization of microstructure, the proposed approach indicates that, as far as elastic response is concerned, the concrete is statistically very close to isotropic material. A more exciting result is again revealed by the proposed approach in plastic hardening and localized failure regimes indicating the sensitivity of computed probability distribution for random variables corresponding to inelastic response macro-scale model parameters.

Three different methods for Bayesian inference have been tested and compared in the proposed approach, with each unknown parameter of reduced model represented as a random field, which can further be reduced to random variables by functional approximation (see [3]). Such a description has two constituents, the measurable function and the measure. One method (MCMC) is identified as updating the measure, whereas the other two methods (EnKF and PceKF) change the measurable function. We formulate both groups of methods as functional approximation of stochastic problems, and introduce especially in combination with the second group of methods a new procedure that does not need any sampling, hence works completely deterministically. It also seems to be the fastest and more reliable when compared with other methods. We have already shown in somewhat simpler setting (see [4]) that it also works for highly nonlinear non-smooth problems with non-Gaussian measures.

Acknowledgements: This work is funded by ANR and DFG (project SELF-TUM).

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Learning in tree-based tensor formats for uncertainty quantification

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Abstract

Uncertainty quantification often meets with high dimensional problems which can also be costly to evaluate. Tensor based methods can be used for the approximation with structured ranks of high dimensional functions. Algorithms are proposed in the framework of supervised statistical learning for learning high dimensional functions in tree based tensor formats which are a particular class of tensor networks [1] whose graphs are dimension partition trees. Such tensor format has a multilinear parametric representation so that the minimization problem introduced to learn the function, solved using an alternating minimisation algorithm, involves a succession of learning problems with linear models. The algorithms provide an approximation with adaptive rank, partition tree and approximation spaces associated to the leaves of the tree [2]. Particular focus will be given to the use and illustration of tree based tensor formats for uncertainty quantification.

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Randomized Model Order Reduction

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Abstract

In this talk we show how randomization can be exploited both for constructing reduced order models and deriving a posteriori error estimators for the approximation error.

We propose local approximation spaces for localized model order reduction procedures such as domain decomposition and multiscale methods, where those spaces are constructed from local solutions of the partial differential equation (PDE) with random boundary conditions [1]. Moreover, we construct reduced models for time-dependent problems by prescribing random initial conditions at random points in time. Extending methods and results from randomized linear algebra [2] allows us to analyze the convergence rate of the randomized approximations.

In addition, we propose a randomized residual-based a posteriori error estimator for reduced order approximations for parametrized PDEs. Here, we address both the error between the reduced and the high fidelity solution [3] and, secondly, between the exact solution of the PDE and any approximation. This error estimator does not require to estimate any stability constants and its effectivity is close to unity with prescribed lower and upper bounds at specified high probability. To derive the estimator we rely on results similar to the restricted isometry property employed in compressed sensing [4].

Work in collaboration with A. Buhr (formerly University of Münster), L. ter Maat (University of Twente), A. T. Patera (Massachusetts Institute of Technology), J. Schleuß (University of Münster) and O. Zahm (University Grenoble Alpes, Inria, CNRS).

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An adaptive hierarchical local HOSVD method.

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Abstract

Tensor methods are one of the main tools to overcome the *curse of dimensionality* when dealing with high dimensional systems. They have been developed and used in several contexts, such as model reduction ([1, 2]) and Uncertainty Quantification ([3]) and proved to be efficient and versatile. However, there are solutions of high-dimensional problems which cannot be written, globally, as a low rank tensor of a given format. Moreover, tensor methods are often intrinsically sequential and when dealing with systems discretised with a large number of degrees of freedom this can lead to a prohibitive computational burden. In the present work, we focus on the HOSVD method ([4]). Instead of looking for a global approximation, a subtensor approximation is defined and computed by means of a greedy method that automatically distributes the approximation error across the subtensors. A hierarchical tree is then adopted in order to adapt the subtensor partition by preserving the approximation accuracy and optimise the storage. This approach is well suited for moderate dimension/large number of degrees of freedom systems and can be efficiently parallelised. Some numerical experiments are proposed to assess the properties of the proposed method.

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From Non-Intrusive model order reduction to advanced real-time engineering

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Abstract

Design space exploration and Parametric FE modeling still the goal to address for the CAE engineer. This dream still unreachable due to the complexity of the problem to solve: give enough flexibility to avoid or reduce the constraint of parametric modeling based on CAD and manage properly the complexity of the parameter combination. On the top of that if you would like to address this problem in real time it will become extremely challenging. In this work, we present an alternative way to address these problems using the commercial software to generate Numerical Charts, or parametric solutions, based on non-intrusive model reduction methods. On the other hand, advanced nonlinear regressions able to proceed in the low-data limit, enabling to accommodate heterogeneous parameters will be proposed and as soon as these parametric expressions will be determined, they can be used for generating large amounts of realizations of the quantity of interest for different choices of the parameters, for supporting data-analytics. Such parametric representations allow the use advanced optimization techniques, evaluate sensitivities and propagate uncertainty all them under the stringent real-time constraint. We will apply this new methodology on several domains of application like Crash, Stamp, Cast, ... to demonstrate the feasibility of this approach with strong nonlinearities. And we will explore the new possible usage of this method to address concurrent engineering in real time across domain to reach the multidomain optimization on line.

Industrial and Medical Digital Twins powered by Reduced Order Modelling

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Abstract

ANSYS research & development is focused on Reduced Order Modelling as a technique to replace very time-consuming 3D calculations by real-time computations giving high fidelity results including fields as velocities on cells or displacements on nodes. Our ROM solutions are integrated in ANSYS Twin Builder which offers creation of digital twins for innovative applications in aeronautics, energy or automotive industries as predictive maintenance and optimal control enriched by sensor data (IoT).

On health care digital twin side, our ROM based real-time solution is the key enabler to develop diagnosis and surgery software which will be part of the clinical protocol. The challenging application of these ROM techniques is to accurately parameterize simulation results with respect to the human variability in terms of anatomy. Various clinical examples in cardiovascular or orthopedics illustrate our innovations.

Application of Proper Generalized Decomposition to Neutron Governing Laws

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Abstract

Governing laws in radiation transport applied to reactor physics include (i) the linear Boltzmann transport equation, in integro-differential advection/reaction equation in a 6-dimensional phase space (position+direction+energy) and (ii) its neutron diffusion approximation, a diffusion/reaction equation in a 4-dimensional phase space (position+energy):

$$\begin{aligned} \vec{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, E, \vec{\Omega}) + \sigma_t(\vec{r}, E) \psi(\vec{r}, E, \vec{\Omega}) \\ = \int_0^\infty dE' \iint_{4\pi} d\Omega' \sigma_s(\vec{r}, E' \rightarrow E, \vec{\Omega}' \cdot \vec{\Omega}) \psi(\vec{r}, E', \vec{\Omega}') + Q(\vec{r}, E, \vec{\Omega}), \end{aligned} \quad (1)$$

and

$$-\vec{\nabla} \cdot D(\vec{r}, E) \cdot \vec{\nabla} \phi(\vec{r}, E) + \sigma_t(\vec{r}, E) \phi(\vec{r}, E) = \int_0^\infty dE' \sigma_s(\vec{r}, E' \rightarrow E) \phi(\vec{r}, E') + Q(\vec{r}, E). \quad (2)$$

In addition, geometries employed in reactor-physics applications typically include heterogeneous domains, often with significant contrast between material properties.

A Proper Generalized Decomposition (PGD) technique is employed over the entire phase-space to solve each of these governing laws. The main highlights of the talk will include :

1. a PGD approach that can efficiently decompose these equations using 1D operators in heterogeneous domains [1];
2. space/energy and space/angle neutronics applications of PGD;
3. a discussion of convergence issues (slow convergence in the number of PGD enrichments) observed in (i) pure advection problems solved with discontinuous Galerkin FEM and (ii) in diffusion/reaction problems with high material contrasts;
4. application of PGD to Uncertainty Quantification for neutronics applications [1].

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Symmetry analysis and equivalence transformations in constitutive modeling

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Abstract

A methodology based on Lie analysis is proposed to investigate the mechanical behavior of materials exhibiting experimental master curves. It is based on the idea that the mechanical response of materials is associated with hidden symmetries reflected in the form of the energy functional and the dissipation potential. In constitutive modeling, symmetry analysis allows condensing the response of materials into so-called master curves - as illustrated by the time-temperature equivalence principle - and to construct rheological models based on a limited number of measurements [1,2,3]. Symmetry methods and conservation law analysis provide invariance properties of BVPs of nonlinear dynamical elasticity. Equivalence transformations extend point symmetries to efficiently reduce the number of significant parameters and thus the computational cost [4]; examples are given for fully nonlinear elastic and viscoelastic BVPs of anisotropic fiber-reinforced solids.

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ABSTRACTS

Day 2 – Thursday, November 21

Afternoon sessions

Coupling PGD model reduction with importance sampling using adaptive subset simulation for reliability analysis

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Abstract

The key point of structural reliability analysis is the estimation of the failure probability. This probability is defined as the integral over the failure domain of a limit state function. Usually, this function is only implicitly given by an underlying finite element simulation. It is generally not possible to compute the failure probability analytically. Therefore, simulation-based methods as well as methods based on surrogate modeling (or response surface methods) have been developed. Nevertheless, these sampling techniques still require a couple of thousand calculations of the underlying finite element model, making reliability analysis computationally expensive for relevant applications.

The aim of this contribution is to increase the efficiency of structural reliability analysis by using the advantages of model reduction techniques, especially the Proper Generalized Decomposition (PGD). Coupling a reduced model with an efficient variance reducing sampling algorithm can reduce the computational cost of reliability analysis drastically.

In the proposed method, an importance sampling technique is coupled with a reduced structural model by means of PGD [1, 2, 3] to estimate the failure probability. Instead of calculating the design point e.g. with optimization algorithms, an estimation of the design point is generated using the idea of subset simulation [4, 5]. The main idea is to decompose the problem in sub-problems with an certain intermediate failure probability. The process is implemented in an adaptive way using an importance sampling method. Furthermore, the PGD approach is used to solve the structural problem a priori for a given parameter space (physical space plus all random parameters of the reliability problem). The PGD approach results in an abacus with all solutions of a system depending on all parameters simultaneously given. This approximation of the solution by a separated form allows a very efficient evaluation of the limit state function in the sampling algorithm.

In the talk, the coupled method is used to estimate the failure probability of examples with different complexity. The increased performance of the proposed coupled method will be shown. In particular, the influence of the discretization error as well as the truncation error of the PGD method will be discussed. A PGD solution algorithm including an adaptive refinement process in each PGD coordinate is presented and the effect on the discretization error will be discussed. Furthermore, the enormous potential in decreasing the computational effort by means of the PGD approach will be exemplarily demonstrated.

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Isogeometric analysis suitable trivariate models generation dedicated to reduced order modeling with geometric parameters

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Abstract

We present a generic framework to construct 3D structured trivariate isogeometric meshes of complicated geometry and arbitrary topology required for reduced order model applications. For that purpose, we use the triangulated solid 3D model's boundary provided from B-Rep CAD (Boundary-Representation in Computer Aided Design) models. The input triangulated mesh is decomposed into a set of cuboids in two steps : pants decomposition and cuboid decomposition [1]. Both segmentations understand the geometry and features of meshes. Cuboid decomposition splits a surface into a set of quadrilateral patches which can define a volumetric layout of the associated boundary surface [2]. Using aligned global parameterization [3], patches of the cuboid decomposition are re-positioned on the surface in a way to achieve low overall distortion, and alignment to principal curvature directions and sharp features. Using the optimized cuboid decomposition, a volumetric layout is extracted. Based on the global parameterization and the structured volumetric layout previously computed, a 3D volumetric parameterization is deduced [4]. For different geometrical instances with the same topology but different geometries, our method allows to have the same representation : 3D trivariate isogeometric isotopological meshes holding the same connectivity. The efficiency of the proposed approach is illustrated through reduced order models with geometric parameters using IGA (IsoGeometric Analysis).

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Physics-guided data-driven reduced-order modeling for nonlinear dynamical problems

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Abstract

Reduced-order modeling for dynamical problems is a difficult task because of stability, expected properties (see [3, 4]) and accuracy issues. Besides that, an popular requirement for ROM building is the non-intrusive feature that does not involve hard intrusive coding into a general purpose FE code. In this communication, we will present a general methodology for non-intrusive ROM building of nonlinear dynamical problems involving two ingredients: i) first the theoretical definition of a ROM structure that returns a general shape of the ROM with nonlinear and interaction terms; ii) second, the use of the (extended) dynamic mode decomposition (DMD and EDMD, [1, 2]) methodology that allows the identification of coefficients into the ROM structure from data produced by a FE code. The methodology will be argued on illustrative cases such as nonlinear heat and fluid-structure interaction problems. We believe that the approach has sufficient generality capability to be applied on a broad spectrum of applications.

Keywords. Dynamical systems; ROM structure; data-driven; DMD; extended DMD; nonlinear observables; non-intrusive approach

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Model reduction in Wasserstein spaces for transport problems

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Abstract

In this talk, we present a new model-order reduction methods for parameter-dependent transport problems using concepts from optimal transport theory, in particular Wasserstein spaces, which provides interesting approximation properties. Some theoretical error bounds will be shown on particular examples, together with preliminary numerical results.

Reducing the input parameter dimension using gradient information

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Abstract

Approximation of multivariate functions is a difficult task when the number of input parameters is large. Identifying the parametric directions where the model does not vary significantly is a key preprocessing step to reduce the dimension of the problem, and thus to facilitate the construction of a reduced order model.

In this talk, we introduce a methodology to detect and exploit such a low-dimensional structure using gradients of the model. The method consists in minimizing an upper-bound of the approximation error obtained using Poincaré-type inequalities [1, 2]. This provides a certified bound on the error caused by the reduction of the parametric dimension.

We then show how the method naturally extends to **nonlinear** dimension reduction, i.e. when the variations of the model is essentially contained on a low-dimensional manifold. Exploiting this kind of low-dimensional structure yields approximations under the form of compositions of functions. Within this framework, there is considerable flexibility and we show on various numerical examples the benefit of this approach.

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A registration method for Model Order Reduction: data compression and geometry reduction

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Abstract

We propose a general — i.e., independent of the underlying equation — registration method for parameterized Model Order Reduction. Given the spatial domain $\Omega \subset \mathbb{R}^d$ and a set of snapshots $\{u^k\}_{k=1}^{n_{\text{train}}}$ over Ω associated with n_{train} values of the model parameters $\mu^1, \dots, \mu^{n_{\text{train}}} \in \mathcal{P}$, the algorithm returns a parameter-dependent bijective mapping $\Phi : \Omega \times \mathcal{P} \rightarrow \mathbb{R}^d$: the mapping is designed to make the mapped manifold $\{u_\mu \circ \Phi_\mu : \mu \in \mathcal{P}\}$ more suited for linear compression methods. We apply the registration procedure, in combination with a linear compression method, to devise low-dimensional representations of solution manifolds with slowly-decaying Kolmogorov N -widths; we also consider the application to problems in parameterized geometries. We present a theoretical result to show the mathematical rigor of the registration procedure. We further present numerical results for several two-dimensional problems, to empirically demonstrate the effectivity of our proposal.

Reduced-order models of nonlinear magneto-quasi-static problems. Alternatives to DEIM-POD?

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Abstract

The use of model order reduction techniques in electromagnetism has boomed in the last decade, and particularly those based on the proper orthogonal decomposition (POD). They have been successfully applied to finite-element type models of electromagnetic devices operating from low to high frequency, e.g. [1, 2]. They have proved high accurate and efficient (extremely low computational cost) if linear materials are involved. In case of nonlinear materials, the application of the discrete empirical interpolation method (DEIM), combined with POD, is widespread and provides excellent results in magnetostatics, e.g. [3]. However, when including eddy-current effects (magneto-quasi-static problems), the DEIM-POD approach lacks robustness, e.g. [4].

This work aims at accounting for saturation and the eddy-current effects *a priori* in order to keep the physical insight that is somehow lost in the blackbox approach of DEIM-POD.

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Reduced Basis Method for the Boussinesq VMS-Smagorinsky model

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Abstract

In this work we present a Boussinesq VMS-Smagorinsky Reduced Basis model applied to a natural convection in a cavity. We consider that the cavity domain is parametrized by a geometrical parameter that modifies the height of the cavity.

With this Reduced Basis method we are able to compute a *real-time* solution of a parametrized PDE by using a Galerkin projection onto a low dimensional space. This low dimensional space is built by certain solutions of the FE problem, called snapshots, selected throughout a Greedy algorithm, where the construction of an *a posteriori* error estimator, developed according with the Brezzi-Rappaz-Raviart theory, becomes essential.

In this Boussinesq model, both the eddy viscosity and eddy conductivity are modeled by a VMS-Smagorinsky setting [1]. These terms are highly non-linear and we need to linearize them by the Empirical Interpolation Method, that allows us to store parameter-independent matrices in the offline phase. Thanks to that, in the online phase we can compute *real-time* solutions.

Finally, we present numerical results developed in FreeFem++ (*cf.* [2]), for which we show the speed-up rate in the computation of the Boussinesq solution, for different geometry configurations of the cavity.

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On the computation of Proper Generalized Decomposition modes of parametric elliptic problems

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Abstract

In a recent paper [1] a new algorithm of Proper Generalized Decomposition for parametric symmetric elliptic partial equation has been introduced. For any given dimension this paper proves the existence of an optimal subspace of at most that dimension which realizes the best approximation -in mean parametric norm associated to the elliptic operator- of the error between the exact solution and the Galerkin solution calculated on the subspace. When the dimension is equal one and making use of a deflation technique to build a series of approximating solutions on finite-dimensional optimal subspaces, the method turns to be a classical progressive proper generalized decomposition. In this contribution we prove the linear convergence of the Power Iterate method applied to compute the modes of the PGD expansion, for symmetric and non-symmetric problems, when the data are small. We also find a spectral convergence ratio of the PGD expansion in the mean parametric norm, for meaningful parametric elliptic problems.

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Numerical analysis on the computation of modes for the Proper Generalized Decomposition to parametric elliptic problems

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Abstract

In this work, we present a numerical analysis and validations on the computation of modes for the Proper Generalized Decomposition (PGD) to parametric elliptic problems, in both the symmetric and non-symmetric cases.

In [1], an iterative deflation algorithm to solve parametric symmetric elliptic equations has been introduced. As the PGD, this algorithm builds a tensorized representation of the parameterized solutions, by means of optimal subspaces that minimize the residual in mean quadratic norm. Each mode of the expansion is computed by minimization of a functional measuring this residual. In this contribution, we introduce a new definition of this functional for non-symmetric problems and prove the linear convergence of the Power Iterate method applied to compute the modes of the PGD expansion, when the data are small. We also find a spectral convergence ratio of the PGD expansion in the mean parametric norm, for meaningful parametric elliptic problems. Finally, we present some numerical test that confirm the theoretical analysis.

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ABSTRACTS

Day 3 – Friday, November 22

Morning sessions

Mechanistic Machine Learning Methods for Mechanical Science and Design/Optimization of Lightweight Material Systems

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Abstract

As in all everyday applications, in engineering problems, the volume of data has increased substantially compared to even a decade ago but analyzing big data is expensive and time-consuming. Data-driven methods, which have been enabled in the past decade by the availability of sensors, data storage, and computational resources, are taking center stage across many disciplines (physical and information) of science. We now have highly scalable solutions for problems in object detection and recognition, machine translation, text-to-speech conversion, recommender systems, and information retrieval. All of these solutions attain state-of-the-art performance when trained with large amounts of data. However, purely data-driven approaches for machine learning present difficulties when the data is scarce and of variable fidelity relative to the complexity of the system.

An open problem in data-driven methods for mechanical science is the efficient and accurate description of heterogeneous material behavior that strongly depends on complex microstructure. To explore the future development and the adaptation of data-driven methods, new mathematical and computational paradigms and broad flexible frameworks are needed, which can lead to probabilistic predictions using the minimum amount of information that can be processed expeditiously and be sufficiently accurate for decision making under uncertainty. Integrating multi-fidelity data into large-scale simulations is necessary to speed up the computation but also to deal with the “hidden physics” not captured by the lack of resolution or the lack of proper constitutive laws or boundary conditions. A number of applications will be presented. A number of material systems applications will be presented.

Learning fluid mechanics from data

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Abstract

We look for a suitable strategy to learn free-surface fluid mechanics from data. In this work we study several learning strategies for fluid sloshing problems based on data. In essence, a reduced-order model of the dynamics of the free surface motion of the fluid is developed under rigorous thermodynamics settings. This model is extracted from data by exploring several strategies. This thermodynamically consistent integrator is developed on the basis of the General Equation for Non-Equilibrium Reversible-Irreversible Coupling, GENERIC [M. Grmela and H.C Oettinger (1997). *Phys. Rev. E.* 56 (6): 6620–6632], framework so as to guarantee the satisfaction of first principles (particularly, the laws of thermodynamics). We show how the resulting method employs a few degrees of freedom, while it allows for a realistic reconstruction of the fluid dynamics of sloshing processes under severe real-time constraints. The proposed method is shown to run faster than real time in a standard laptop.

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Perspectives in Reduced Order Methods in Computational Fluid Dynamics: the effort of increasing the Reynolds number

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Abstract

We provide some perspectives for Reduced Order Methods (ROM) of parametric Partial Differential Equations (PDEs) with a special interest in parametric flow (control) problems arising in offline-online Computational Fluid Dynamics (CFD).

Efficient parametrizations (random inputs, geometry, physics) are very important to be able to properly address an offline-online decoupling of the computational procedures and to allow competitive computational performances. Current ROM developments in CFD include: a better use of stable high fidelity methods, considering also spectral element method and finite volume discretizations, to enhance the quality of the reduced model and the capability to increase Reynolds number, including turbulent patterns; more efficient sampling techniques to reduce the number of the basis functions, retained as snapshots, as well as the dimension of online systems; the improvements of the certification of accuracy based on residual based error bounds and on the stability factors, as well as the the guarantee of the stability of the approximation with proper space enrichments. For nonlinear systems also the investigation on bifurcations of parametric solutions is crucial and it may be obtained thanks to a reduced eigenvalue analysis of the linearised operator. All the previous aspects are very important in CFD problems to focus in real time on complex parametric industrial, environmental and biomedical flow control problems. Model flow problems will focus on few benchmarks, as well as examples of applications concerning shape optimisation applied to industrial problems.

Physical ROMs: how to accomodate nonparametrized variability, nonintrusivity, performance and error indication for large scale industrial applications?

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Abstract

Physical reduced order modeling has been a subject of active interest since the opening of Safran group's corporate reseach center 4 years ago. To be applied to industrial settings, reduced order models (ROMs) must be nonlinear, scalable, robust and able to handle complex models and nonparametrized variability. Other important desirable properties are nonintrusivity (the ability to construct ROMs from snapshots generated by commercial software) and error indication.

In this talk, we present some of Safran's contributions to the aforementioned features of physical reduced order modeling and illustrate them in large scale industrial settings. Scalable nonintrusive physical ROMs in nonlinear structural mechanics with elastoviscoplastic material, error indication and nonparametrized variability are developed in [1, 2]. Scalable physical ROMs in fluid dynamics for the incompressible and turbulent Navier Stokes and nonparametrized geometrical variability are developed in [3, 4].

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A two-scale FE² method using neural networks

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Abstract

In this work, a two-scale (FE²-like) [1, 2] computational homogenization method based on neural networks is proposed. In classical multiscale FE² approaches [3], the macroscale structure calculation requires solving at each Gauss point of the macro mesh a nonlinear problem on a Representative Volume Element (RVE), for each load step and each iteration of the macro Newton algorithm. In this work, we propose to replace the costly nonlinear RVE calculations by a surrogate model of the nonlinear behavior based on Neural Networks (NN). In this context, off-line calculations on RVEs are performed and a surface response of the effective quantities such as macro stress, macro fluxes are constructed automatically as a function of the full 3D macroscopic load (macro strains, macro gradients, etc.). As a result, drastic decrease of the computational costs are achieved in FE² procedures, as the cost of RVE nonlinear solving is reported to off-line calculations. Another appealing feature of this procedure is that the NN-model of the macroscopic response can also include the influence of microscopic parameters (volume fraction, local material parameters, etc.), opening route for material optimization and design in a nonlinear context, which was a tedious task with available approaches. The method is applied to both nonlinear hyperelastic composites and nonlinear electric composites structure calculations.

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Development of reduced structural theories for composite plates and shells via machine learning

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Abstract

This paper presents a new approach for the development of structural models via three well-established frameworks, namely, the Carrera Unified Formulation (CUF)[1], the Axiomatic/Asymptotic Method (AAM)[2], and Artificial Neural Networks (NN)[3]. CUF and AAM provide the finite element arrays and measure the relevance of any given generalized displacement variable. The NN training makes use of the data from CUF-AAM and the outputs are the Best Theory Diagrams [4] - curves providing the minimum number of nodal degrees of freedom required to satisfy a given accuracy requirement - and the accuracy of any structural theory. The main governing equations for plate and shell finite elements via CUF are the following and lead to the implementation of any order theory,

$$\mathbf{u}(x, y, z) = F_{\tau} N_i(z) \mathbf{u}_{\tau i}(x, y) \Rightarrow \int_{\Omega_k} \int_{A_k} (\delta \boldsymbol{\epsilon}^k T \boldsymbol{\sigma}^k + \rho^k \delta \mathbf{u}^k T \ddot{\mathbf{u}}^k) H_{\alpha}^k H_{\beta}^k d\Omega_k dz = 0 \Rightarrow \mathbf{m}_{\tau i s j}^k \ddot{\mathbf{u}}_{\tau i}^k + \mathbf{k}_{\tau s i j}^k \mathbf{u}_{\tau i}^k = 0 \quad (1)$$

The inputs of the NN are combinations of the fifteen generalized displacement variables of a fourth-order model and the thickness ratio,

$$\begin{aligned} u_x &= u_{x_1} + z u_{x_2} + z^4 u_{x_5} \\ u_y &= u_{y_1} + z u_{y_2} + z^3 u_{y_4}, \quad h/a = 0.1, \Rightarrow [11111110010101000.1] \\ u_z &= u_{z_1} + z u_{z_2} + z^2 u_{z_3} \end{aligned} \quad (2)$$

Where '1' indicates an active variable and '0' a deactivated one. The targets for the NN training are the errors over the first natural frequencies,

$$Error = \sum_{i=1}^{10} \frac{f_i / f_i^{N=4}}{10} \quad (3)$$

Where the reference frequencies are those by the full fourth-order model. The NN configuration is a multilayer feed-forward with early stopping and mean squared error as the objective function. Each layer has ten neurons. This paper adopts Levenberg-Marquardt training functions. The numerical results refer to a 0/90/0 square simply-supported spherical panel as in [4], R/a = 5. Figure 1 shows the BTD computed via two different approaches. FE refers to the full finite element approach considering 2¹⁵ modal analysis, i.e., one per each model stemming from the combinations of the fifteen terms of the fourth-order model. The BTD reports models that, for a given number of degrees of freedom

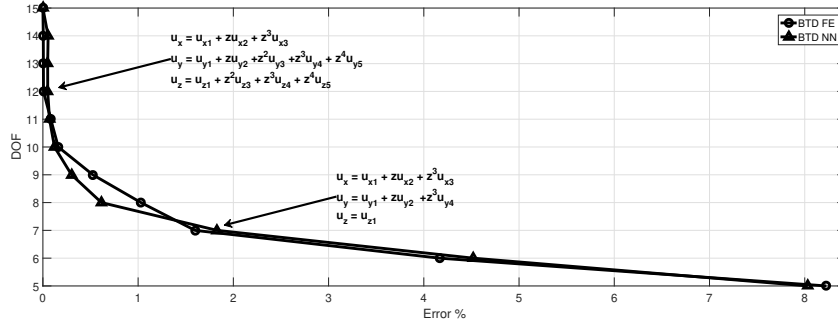


Figure 1: BTD from FE and NN $a/h = 5$.

(DOF), provides the minimum error. NN refers to the BTD obtained from a trained neural network. Training considered $a/h = 10$ and $a/h = 2$ and a population of some 2000 structural models. The plot reports the explicit displacement field of the 12 and 7 DOF models. Table 1 reports the error over the first ten frequencies as computed by the FE analysis and by the trained NN. PTD refers to a displacement field having full linear expansion and a parabolic term on the transverse displacement. TSDT is a model having full third-order expansions over the in-plane displacements and constant transverse one. The results suggest that

Model	DOF	FE	NN
FSDT	5	13.4	13.3
PTD	7	12.9	10.0
TSDT	9	2.6	2.0

Table 1: Mean error (%) on the first ten frequencies via FE and NN, $a/h = 5$.

- The use of NN is promising as a tool to evaluate the accuracy of structural theories with very high computational efficiency. Also, as the network is trained considering physical features such as the thickness, it may provide good estimates for different values of the same feature.
- Considering the structural models, the results show that, for the problem considered, models from the literature may fail in detecting the first natural frequencies with sufficient accuracy. As a general guideline, and as known from the literature, third-order in-plane variables are decisive.

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Nonlinear dimensionality reduction of parametrized PDEs by reduced basis methods and deep learning techniques

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Abstract

We present state-of-art projection-based reduced order models (ROMs) for parametrized PDEs, with special emphasis on the reduced basis method for nonlinear time-dependent problems. We propose new strategies to construct ROMs exploiting deep learning techniques, such as convolutional neural networks. Both the construction of a reduced-order space and the solution of the resulting reduced-order problem can be designed by exploiting neural networks for the sake of computational efficiency. Applications of interest deal with problems featuring highly nonlinear solution manifolds, as well as strong dependence of the solution on parameters, and requiring higher spatio-temporal accuracy; a relevant example is provided by coupled problems related with cardiac electrophysiology, whose goal is modeling the propagation of electric potentials in the cardiac muscle. In this case, suitable combinations of (physics-based) projection-based ROMs and (data-driven) deep/machine learning techniques can provide remarkable computational savings without affecting numerical accuracy substantially.

Nonintrusive reduced basis methods through machine learning

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Abstract

The development of reduced order models for complex applications, offering the promise for rapid and accurate evaluation of the output of complex models under parameterized variation, remains a very active research area. Applications are found in problems which require many evaluations, sampled over a potentially large parameter space, such as in optimization, control, uncertainty quantification and applications where near real-time response is needed.

However, many challenges remain to secure the flexibility, robustness, and efficiency needed for general large-scale applications, in particular for nonlinear and/or time-dependent problems.

In this talk we discuss the use techniques from machine learning, eg Gaussian process regression and neural networks, to facilitate the development of efficient and accurate non-intrusive reduced order models. We shall offer a brief introduction to the different techniques, including ideas for greedy regression/active learning and error estimation, and illustrate the performance by examples taken from nonlinear mechanics and fluid dynamics.

To enable the modeling of more complex problems, we discuss the development of hybrid element based reduced order model with nonlinear elements, allowing for the rapid and fast evaluation of large and complex structures for design and uncertainty quantification applications.

Time permitting we extend the discussion to the multi-fidelity case where different models are combined through cokriging.

ABSTRACTS

Day 2 – Thursday, November 21

Posters evening session

Low-order feature extraction technique and unsupervised learning for SHM under high-dimensional data

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Abstract

Data-driven damage localization is a demanding process for vibration-based structural health monitoring (SHM) strategies. The ability to locate single and multiple damage states featuring different severity levels, particularly the smaller ones, plays a prominent role in establishing an effective and robust method for damage assessment. The statistical pattern recognition paradigm based on feature extraction and statistical decision-making can be a successful framework for this process [1,2]. In case of data gathered by dense sensor networks, which provide vibration datasets of high dimensionality and large volume, this framework may be time-consuming or complex; it also results questionable whether existing order selection techniques are able to provide a low-order feature extraction approach [1]. Furthermore, a fast decision-making process within an unsupervised learning strategy can help overcome the main obstacles experienced in the accurate localization of damage, if large volumes of damage-sensitive features are extracted from high-dimensional samples gathered by the mentioned dense sensor networks. In this study, we propose an efficient unsupervised learning method for feature extraction by an iterative approach based on order reduction in AutoRegressive (AR) modelling [3], and for damage localization through a statistical distance method termed Kullback-Leibler Divergence with Empirical Probability Measure (KLDEPM).

The proposed iterative feature extraction approach mainly aims at model order reduction. In the training phase, this is accomplished simultaneously with parameter estimation and residual extraction; in the following monitoring phase, the low-order AR model is adopted to only extract the new residuals. With this approach, correlated residual samples of the AR model are adopted as a new time series dataset to reduce the model order at each iteration. This strategy is shown to effectively and efficiently reduce the order of the AR model necessary for feature extraction. Selecting a reduced AR order that guarantees model sufficiency and accuracy by generating uncorrelated residuals is

therefore the main strength of the proposed order selection method; in case of an inadequate order selection, the model may not be able to capture the underlying dynamics of the structure, and lead to the extraction of features insensitive to damage.

The proposed KLDEPM, which is an enhancement of the classical KLD technique, next exploits a segmentation technique to subdivide the handled random data (i.e. the AR model residuals relevant to the virgin and damaged conditions) into independent segments, and compute a distance between the features relevant to the training stage and those relevant to the monitoring stage, based on the theory of empirical probability measure. This procedure provides an effective distance approach for damage identification and a fast tool for decision-making. To establish an unsupervised learning strategy, a threshold limit is determined by computing the mean of the 95% confidence intervals of distance quantities obtained from the virgin conditions in the training phase. The sensor location(s) associated with the KLDEPM value(s) greater than the threshold limit is (are) identified as the damaged area(s) of the structure.

Numerical concrete beam and IASC-ASCE experimental benchmarks are considered to assess the accuracy of the proposed SHM method, and the improvement in the performance (in terms of feature extraction and damage localization) against alternative approaches available in the literature. More specifically, the proposed order selection method aimed at reducing the model order is compared with the state-of-the-art Bayesian Information Criterion technique, and with a conventional residual-based feature extraction approach. Furthermore, the method is benchmarked by the classical KLD technique and the Kolmogorov–Smirnov test. Results demonstrate that both the iterative feature extraction technique and the KDLEPM method are superior to their counterparts, and provide fast unsupervised learning strategies to extract reliable damage-sensitive features and locate single and multiple damages of different severities.

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Application of Code2Vect-S-PGD in an industrial framework: Acoustic prototypes.

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Abstract

The industry needs to adapt itself to a growing market in which innovative technological solutions must be continuously included. Moreover, in order to be competitive, these new technologies have to be fastly developed and introduced. The development of new prototypes goes through several stages: a prototype is designed, a sample is fabricated and tested until it converges to the required solution, and then the selected design is tested in big scale. All these stages take time to fulfill and the use of Model order reduction techniques combined with artificial intelligence comes in handy for alleviating these time and thus contributing to a faster implementation of new technologies.

In that framework a new algorithm was developed, Code2Vect [1], enabling the classification and regression in the low-data limit. In this work we present an optimization of the algorithm based on the implementation of the Sparse-PGD [2], which will improve the algorithm capabilities for working in high-dimensional spaces. Moreover we will implement the hybrid-twinTM technique developed by ESI group in an industrial framework regarding the development of acoustic prototypes. To show the regression capabilities of the algorithm data from industrial test-samples will be analyzed in order to find a regression model that will enable a reduction on the required samples, thus saving money and time.

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Model order reduction for a kinetic Fokker-Planck equation

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Abstract

In this talk we present model reduction techniques in the context of a well-posed variational formulation and stable discretization for a kinetic Fokker-Planck equation.

The equation describes particle densities in phase space consisting of temporal, spatial, and velocity variables, and is used for instance to model tumor growth on the mesoscopic level. The structure of the equation poses challenges for the theoretical framework of the exact solution as well as the numerical realization of approximations.

Since the equation contains diffusive velocity variables as well as advective space and time variables, we cannot use standard theory for parabolic or hyperbolic equations to prove well-posedness of the problem. Instead, we obtain a variational formulation on the full space-time-velocity domain by combining formulations for parabolic equations as defined e.g. in [2] and for first order transport equations [1].

Based on this setting, we develop a suitable stable discretization. However, due to the high-dimensionality of the equation, we cannot simply use standard finite element methods for the whole problem. Instead, we use a tensor-based approach. In detail, we develop a Hierarchical Model Reduction approach, deriving problem-adapted (low-dimensional) bases in the velocity domain. Here, we build on our previous work for the one-dimensional Fokker-Planck equation [3].

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Reduced Basis Method applied to the Smagorinsky Model

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Abstract

We are interested in the inclusion of the courtyards into the building design, since this kind of structure is very interesting for the regulation of the temperature inside the building. Unfortunately, the tools that the industry uses are not catching the benefits of including the courtyards into the structure of the building. For this reason, we are studying the air that flows into the courtyard. This involves computing models for several parameters which could be geometrical or physical. Sometimes, each computation could take a long time and this situation is not the most desirable.

This reason encourages us to consider the Basis Reduced Method that allow us to obtain a faster solution with a little error. To this purpose, we need to develop *a posteriori* error bound estimator for the error between the high fidelity solution (in this case, we will consider the Finite Element Solution) and the Reduced Basis Solution.

In this talk, we show how the *a posteriori* error bound estimator is built according with the Brezzi-Raviart-Rappaz theory and computed applied to the Smagorinsky Model.

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A Proper Generalized Decomposition approach with Inertia Relief (PGD-IR) for the static analysis of geometrically parametrized unconstrained structures

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Abstract

Unconstrained structures are widespread in the automotive and aerospace industry. As is well known, due to the singularity of the stiffness matrix, conventional static analyses cannot be performed if boundary conditions are not imposed. At the same time, imposing dummy constraints in order to make a free-body system statically determinate leads to unrealistic reaction forces and, as a consequence, an incorrect distribution of the internal stresses. The inertia relief (IR) approach [1] is widely used by industry in this context. The main idea is to counteract the applied loads by a set of rigid body accelerations, the latter providing body forces which are distributed over the structure in such a way that the total sum of the applied forces on the structure is zero and the static analysis can be performed.

In this work, the IR analysis is embedded in the context of a new car development process. A very crucial task during this phase is the design of components, which must be continuously re-adapted in order to meet all the established target specifications. Obviously, any change in the material and geometrical parameters might have a considerable effect on the static and dynamic global stiffness of the structure. The ability to perform an accurate and fast evaluation of all possible configurations is still an open challenge, due to the high computational cost of each simulation involved. A solution to decrease the numerical complexity of high-dimensional problems is to introduce reduced order models (ROMs) [2].

The novelty of this work is the implementation of the IR method in a ROM framework for the solution of a geometrical parametric problem. In particular, the so-called Proper Generalized Decomposition Least-Squares approximation [3] is proposed as a numerical tool able to provide an offline explicit separable solution in terms of an a-priori unknown number of parametric and mechanic modes. An unconstrained structure with material and geometrical parameters is tested by means of the proposed PGD-IR approach to show how the method ensures sensitivity to parameter variations and allows for very fast queries post-process. Moreover, unlike many other ROM methods, the proposed approach does not require any pre-processing phase to collect prior knowledge of the solution. The final goal of this project is to devise a computational tool which can guide the designer in the decision making, so that the impact of certain design parameters on the comfort static and dynamic car body stiffness can be effectively considered already in the preliminary steps.

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Characterization of insulating materials by implementation of a reduced order model in the inverse procedure

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Abstract

Nowadays, regulatory measures have been taken to help reduce energy consumption impact on the global warming issue. One of them is to produce high performing materials for the construction sector. Insulating materials being one of the most affected due to their composition, and their low environmental performance [1]. An existing alternative, the bio-based insulators, address both environmental and thermal criteria are becoming an attractive solution to the lower the carbon emissions.

However, since they present dynamic properties and aren't homogenous, their characterization is somewhat inaccurate and existing characterization methods neglect other intrinsic phenomena. This paper propose to adapt a laboratory-built probe and replace its analytical method with an iterative process using a reduction technique in order to maintain the complexity of the geometry but still achieve a rather fast identification compared to other numerical time-consuming calculations.

The modal reduction technique applied here is the AROMM method, extension of the BERM method [2,3]. The main difficulty is presented when building the basis for the problem, since the thermal properties of the soon-to-be characterize material are unknown. In order to overcome this, a set of reference temperatures was set, being able to englobe a larger sample of insulating materials. In this way, the resulting reduced model gives positive and performing results for whatever insulating material tested [see Fig.1]. This models are integrated inside an inverse iterative process that uses the Particle Swarm Optimization method. Results have proven to be satisfactory with this approach [see Fig.2].

Future studies are contemplated to validating the numerical characterization with a real measurement taken from a sample. Furthermore, a possible scenario is to adapt the procedure to be used for on-site characterization, since portability and speed play a key role for construction materials.

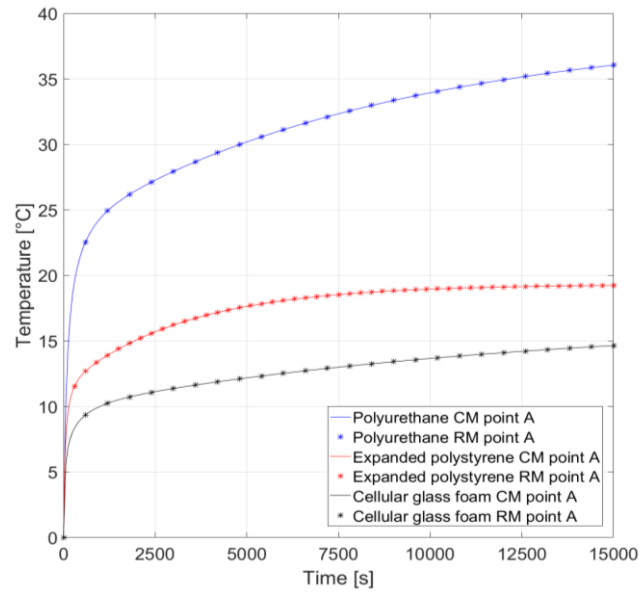


Figure 1 – Temperature evolution comparison

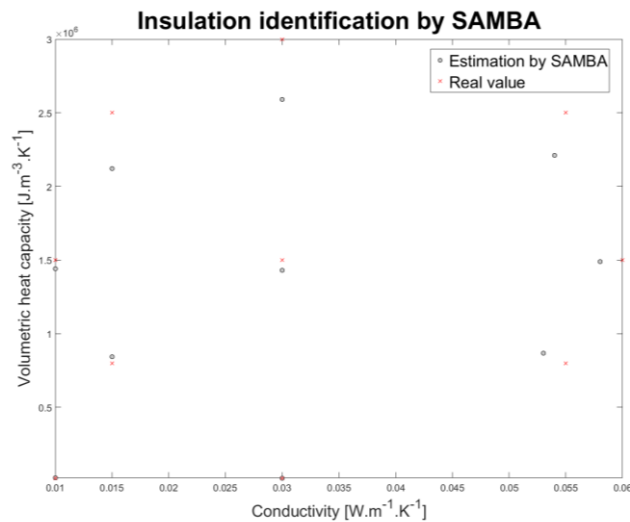


Figure 2 – Identification results

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Adaptive parameter sampling for the Reduced Basis method

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Abstract

For dynamical systems with high-dimensional parameter spaces, the offline time of the Reduced Basis (RB) method can become prohibitively large [1]. This is because the error estimator needs to be evaluated for all the parameters in a training set at every iteration of the greedy algorithm, until convergence. It is known that (D)EIM ((Discrete) Empirical Interpolation Method) [2, 3] is used to reduce the computational complexity of the nonlinearity of the reduced-order model (ROM). In this work, we explore a new application of (D)EIM, where the parameters of importance can be selected through the (D)EIM procedure. We focus on dynamical systems with an output of interest. For such systems, we propose a two-stage algorithm to control the complexity of the training set. In the first stage, we perform a coarse RB approximation of the problem using a fine training set. In the second stage, the cardinality of a coarse training set shall be user-defined. The coarse training set only includes parameters picked by performing (D)EIM approximation to the output snapshots matrix computed from the coarse reduced order model at all the parameters in the fine training set. Furthermore, we add a fixed number of randomly sampled parameters in order to promote a full exploration of the parameter space. We show through numerical examples that this two stage approximation is able to select the most important parameters to the coarse training set. An independent set of test parameters is used to verify that the reduced order model obtained through this adaptive sampling approach indeed meets the required tolerance and accelerates the offline computation.

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On model reduction applied to a Trefftz method to simulate the transient dynamic response resulting from a shock

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Abstract

For a number of industries, such the ArianeGroup launchers, vibration simulation is a crucial step in the conception. Tests on structures subjected to pyrotechnic shocks show that medium frequencies cannot be neglected. The challenge is to compute the transient response of these shocks including nonlinearity and medium frequencies. The most common method for vibration simulation is the standard Finite Element Method (FEM), but this method becomes too expensive in time if mid- and high-frequency phenomena are to be properly taken into account. An alternative are the wave-based approaches, such as the Variational Theory of Complex Rays (VTCR) [1]. This Trefftz method effectively solves linear problems such as plate and shell assemblies in the mid-frequency range [2]. The construction of the solution in time requires a resolution over the entire frequency domain, which implies prohibitive calculation costs. Attempts at model reduction approaches have been proposed in the past to reduce these costs [3] without achieving the desired performance. Here we propose a more efficient and robust method. It is based on a VTCR conditioning technique, and on the use of the artificial damping technique [4]. Illustrations on 2D acoustic cases will be given.

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Distortion of a part during Selective Laser Melting using the Proper Generalized Decomposition

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Abstract

Additive manufacturing shapes the future of manufacturing by offering flexibility and efficiency, material wise and economic wise. Therefore, active research in this field aims at overcoming potential defects and optimizing the final quality of the printed part. Selective Laser Melting is one of the additive manufacturing processes which operates at high temperatures, therefore enhancing the homogeneity between the printed layers. However with high temperature gradients comes high distortion in the final part.

In this work, we present a technique to evaluate the distortions caused by temperature gradients during cooling using the proper generalized decomposition [1]. This method assumes that the shrinkage due to cooling is a function of the parametrized printing trajectories [2] and hence calculates the thermal body forces accordingly. An in-plane-out-of-plane decomposition by the means of the PGD is used to obtain the final part distortion efficiently. This method is applied on different printing trajectories and its results will be presented. In addition, predicting the final distortion offers the availability to compensate it "a priori", before printing, and hence obtaining the desired tolerances.

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Model order reduction strategy for fatigue damage analysis

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Abstract

Fatigue analysis based on continuum damage mechanics requires advanced numerical strategies due to the highly non-linear behaviour and the possible high number of involved time steps. A numerical strategy based on LATIN-PGD has been proposed to tackle efficiently fatigue damage [1]. The approach has been extended to tackle high number of cycles including a reduced approach in time [2], fatigue regime for which the material behaves elastically at the macroscopic scale, whereas damage is governed by micro-plasticity [3], and random loading while keeping a small size of the reduced basis [4].

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Modal reduction adapted to heat transfer with radiation in a furnace

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Abstract

Numerical thermal studies of industrial applications such as climatic chambers, satellites or furnaces, require a numerical approach with radiative heat transfer and a fine mesh. Because of the specificity of thermal radiation (integration into space and strong non linearity), computation time of such problem becomes unbearably long and the use of modal reduced models are particularly interesting.

In the context of model reduction, we present here an adaptation of the Amalgam Reduced Order Modal Model (AROMM) method for heat radiation and conduction problems in an enclosure. AROMM method is an extension of the BERM method [1, 2, 3].

The physical problem under study is the thermal treatment of a titanium object placed on a stand in a furnace. The object is heated by two radiant tubes ($673K < T_{tube} < 1273K$) equipped with parabolic reflectors (see Fig. 1 and Fig. 2).

Three scenarios characterised by different boundary conditions are presented. The AROMM method, with a reduced modal model order of 100, exhibits for each scenario a computation time over 200 time faster than a finite elements model while the relative maximal error is lower than 7% and the mean error below 0.2%. These good results open the way for control or identification procedure with radiation.

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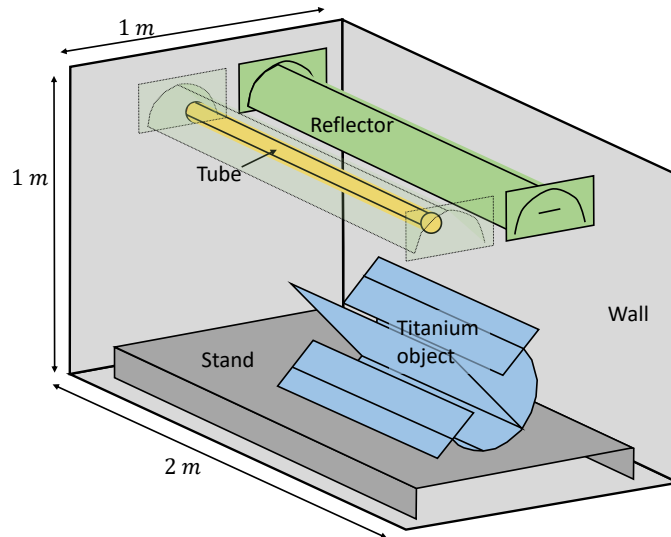
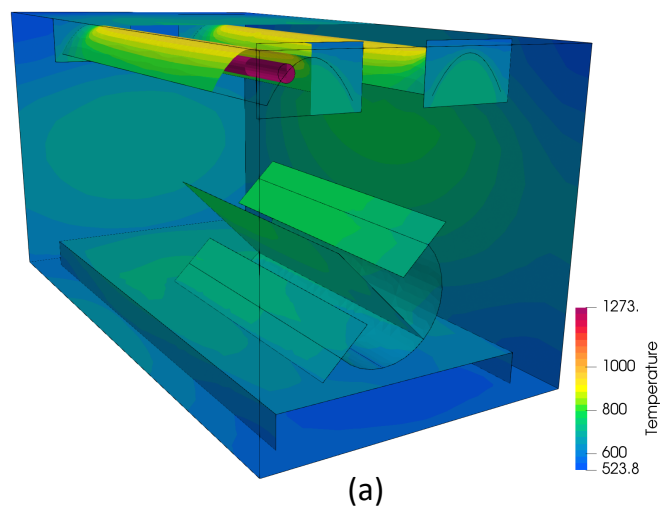
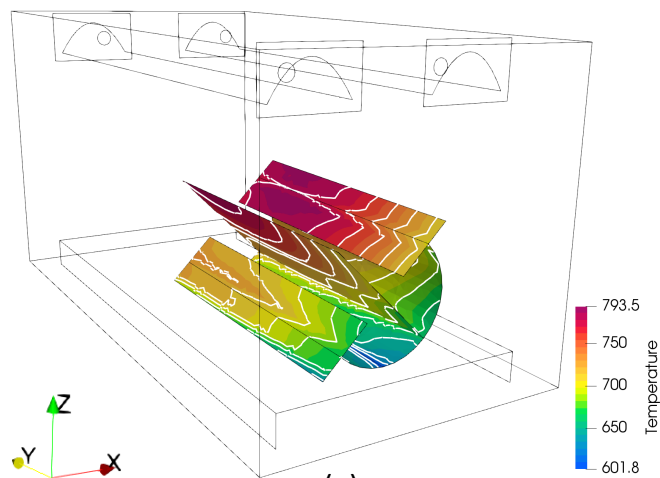


Figure 1: The considered geometry



(a)



(a)

Figure 2: Temperature fields for scenario 3 at time 3 000 s. Figure (a) represents the complete thermal scene, whereas Figure (b) focuses on titanium item

New Tools for Data-Driven Computation in Nonlinear Solid Mechanics

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Abstract

Through the recent revolution of Big Data, data-driven computation tools should become predominant, using large volumes of data which cover densely the area of interest. Pioneering works are due to Ortiz and Chinesta and their teams [1], [2],[3]. Although their approaches tend to minimise the use of the current knowledge in Materials Science and treat only special situations with rather simple behaviours, they proved very efficient in these situations.

The proposed approach, which relies heavily on Materials Science, is based on the internal variables approach, widely used to describe the material state. The first difficulty resides in the structuring step, where hidden variables must be rebuilt from the raw data, without knowing them a priori, in order to obtain the Experimental Constitutive Manifold (ECM). Such a task is not feasible with the classical Big Data methods ([4]), and new tools, are proposed to do this structuring ([5]). Then, classical reduction technics can be used to reduce the amount of data by suppressing redundancies.

The data-driven calculation part relies on the use of the Latin-PGD solver, which separates constitutive equations and admissibility equations, is the ideal solver since only the constitutive equations are impacted by the a priori unknown material properties and can be replaced by the discrete ECM.

The proposed approach has been implemented and tested for a viscoplastic material model with internal variables, as a proof of concept.

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Hyperreduction of Nonlinear Petrov-Galerkin Reduced-Order Models Using ECSW

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Abstract

To extract the desired computational speed-up from nonlinear projection-based reduced-order models (PROMs), a second layer of approximations is typically required after the original subspace approximation is performed using a reduced-order basis (ROB). This step, known as hyperreduction, seeks to accelerate specifically the evaluation of the projected nonlinear quantities using an approximate approach whose computational complexity scales only with the dimension of the PROM. All hyperreduction methods that have been applied so far to CFD problems are of the approximate-then-project type –that is, they approximate first the quantity to be projected on the left ROB, then compute its exact projection. They typically rely on an approach that was first developed for the gappy proper orthogonal decomposition, or gappy POD, method. Unfortunately, the implementation of most, if not all, of these hyperreduction methods requires a number of user-specified parameters that are hard to determine. As such, they are less desirable from a practical viewpoint. Alternatives to this first class of hyperreduction methods are of the project-then-approximate type, where the projected nonlinear quantities are directly approximated. The energy-conserving sampling and weighting (ECSW) method [1] is a recent demonstration of the ability of this class of hyperreduction methods to produce stable and accurate hyperreduced Galerkin PROMs for second-order hyperbolic problems. ECSW relies on an approximate, data-driven, generalized quadrature rule determined via the solution offline of an optimization problem for which several distributed solution algorithms have been studied [2]. In this work, a quadrature-based method for hyperreduction of the project-then-approximate type inspired by ECSW is presented for first-order hyperbolic Petrov-Galerkin PROMs, such as those designed for the reduction of nonlinear CFD models. This proposed ECSW-like method represents an improvement of the state of the art of hyperreduction techniques for CFD problems. Its accuracy and computational efficiency will be demonstrated in this poster for a variety of RANS and LES computations associated with complex geometries.

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Tensor Model Learner

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Abstract

The conception of new designs where topology, material parameters or boundary conditions is of crucial interest in many industrial applications. Surrogate modeling has been a natural ally to create response surfaces allowing decision making under real time constraints. Among these techniques, Artificial Intelligence (AI) algorithms have been successfully implemented in a wide variety of applications, for instance, CNN [1] are very efficient for image recognition just like LSTM for time series reconstruction.

However, it is important to reckon that there exists to main bottlenecks related to the training phase of most IA algorithms. The first one is the excessive amount of data samples, sometimes unaccessible, required to obtain a good model accuracy. The second difficulty involves the training cost, as it tends to increase whenever the complexity of the neural network increases.

In order to circumvent, or in any case alleviate, these drawbacks, it is convenient to revisit the concepts such as separated representations or Greedy enrichments, which are already present in Model Order Reduction [2] (MOR) techniques. Merging this concepts within the AI algorithms might open new routes to improve the architecture of the neural networks.

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A modal substructuring method for non-conformal mesh.

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Abstract

Component Modes Synthesis involves dividing the system in elementary entities (the sub-structures), calculating the modes for each one of these entities and finally assembling them to solve the initial problem. Component Modes Synthesis is widely used in structural mechanics since the pioneering work of Hurty [1] and Craig & Bampton [2]. This communication investigates sub-structures with non-conformal mesh at their common interface. The problem of junction on non-conformal meshes has been studied in finite elements. Among the different methods, mortar method has been introduced by Bernardi *et al.* [4] for the resolution of 2D elliptic partial differential equations. In this method one of the two borders situated on each side of the interface is chosen as a mortar border, the other one being labeled as non-mortar. The interest of partitioning the interface this way is indissociable from the second point of the method : to ensure the continuity of the solution through the interface, the continuity conditions are not enforced point by point, but in a weak sense. We propose in this communication an adaptation of the mortar method to modal formulation.

Let $\Omega^{(1)}$ and $\Omega^{(2)}$ two domains separated by their common boundary Γ , such that $\Omega = \Omega^{(1)} \cup \Omega^{(2)}$ and $\Omega^{(1)} \cap \Omega^{(2)} = \emptyset$. We denote $\Gamma = \overline{\Omega^{(1)}} \cap \overline{\Omega^{(2)}}$ and $\Gamma^{1 \rightarrow 2} = \overline{\Omega^{(1)}} \cap \Gamma$ the interface “on the side of $\Omega^{(1)}$ ”. Finally, the boundary with the surroundings is noted $\Sigma^{(k)} = \partial\Omega^{(k)} \cap \partial\Omega$ and $\underline{n}_{k,l}$ is the normal vector to $\Omega^{(k)}$ on Γ , $k, l \in \{1, 2\}$, $k \neq l$, while \underline{n}_k is the normal to $\Omega^{(k)}$ on $\Sigma^{(k)}$. Both sub-domains exchange heat with the surroundings via a global heat transfer coefficient h , and can be subjected to an internal heat power dissipation ϖ . If we denote φ the heat flux density $\underline{\kappa}^{(1)} \cdot \underline{\nabla} T^{(1)}$ at the interface, the variational formulation of the heat equation writes:

Definition 0.1 Find $(T^{(1)}, T^{(2)}, \varphi) \in H^1(\Omega^{(1)}) \times H^1(\Omega^{(2)}) \times W(\Gamma)$ such that

$$\begin{aligned} \sum_{k=1,2} \int_{\Omega^{(k)}} C^{(k)} \frac{\partial T^{(k)}}{\partial t} v^{(k)} + \sum_{k=1,2} \int_{\Omega^{(k)}} \underline{\nabla} T^{(k)} \cdot \underline{\kappa}^{(k)} \cdot \underline{\nabla} v^{(k)} + \\ \sum_{k=1,2} \int_{\Sigma^{(k)}} h^{(k)} T^{(k)} v^{(k)} + \int_{\Gamma^{(2,1)}} \varphi \llbracket v \rrbracket_{\Gamma^{2 \rightarrow 1}} \\ = \sum_{k=1,2} \int_{\Omega^{(k)}} \varpi^{(k)} v^{(k)} + \sum_{k=1,2} \int_{\Sigma^{(k)}} h^{(k)} T_{ext} v^{(k)} \end{aligned} \quad (1a)$$

$$\int_{\Gamma^{2 \rightarrow 1}} \llbracket T \rrbracket_{\Gamma^{2 \rightarrow 1}} w = 0 \quad (1b)$$

for all $(v^{(1)}, v^{(2)}, w) \in H^1(\Omega^{(1)}) \times H^1(\Omega^{(2)}) \times W(\Gamma)$, where $W(\Gamma)$ is a functional space to be defined.

The temperature is decomposed on a Dirichlet-Steklov base:

$$\begin{array}{l|l} \Omega & -\underline{\nabla} \cdot \underline{\kappa} \cdot \underline{\nabla} V^D = \lambda^D c V^D \\ \partial\Omega & V^D = 0 \end{array} \quad \left| \quad \begin{array}{l} \underline{\nabla} \cdot \underline{\kappa} \cdot \underline{\nabla} V^S = 0 \\ \underline{\kappa} \cdot \underline{\nabla} V^S \cdot \underline{n} = \lambda^S \zeta(\underline{x}) V^S \end{array} \right. \quad (2)$$

Since the Dirichlet modes are insufficient to reconstruct the solution on the boundary, the Steklov modes are added to them, which enables to reconstruct the heterogeneity of the boundary conditions. Temperature is searched as:

$$T(\underline{x}, t) = \sum_i V_i^D(\underline{x}) x_i^D(t) + \sum_j V_j^S(\underline{x}) x_j^S(t) \quad (3)$$

The originality of our method is to also decompose the heat flux on a modal base:

$$\varphi = \sum_i \varphi_i \phi_i(\underline{x}) \quad (4)$$

Several possibilities exist for functions $\phi_i(\underline{x})$. In the bi-dimensional case, the normal derivative of mortar functions (which are 1D) is zero on the boundaries of the interface. Following that idea, we decide to decompose the heat flux on a Neumann base :

$$-\nabla^2 \phi = \lambda \phi \quad \text{sur } \Gamma \quad | \quad \underline{\nabla} \phi \cdot \underline{n} = 0 \quad \text{sur } \partial\Gamma \quad (5)$$

where $\partial\Gamma$ is the boundary of the interface.

A realistic example illustrates the method : a library of electronic components reduced models is built, then used to create two electronic cards. Results meet high expectations : the substructured reduced model is 2 500 times faster to solve than a finite elements one, while keeping a precision below $1^\circ C$ on the critical parameters.

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Proper orthogonal decomposition with an adaptive selection of modes for problems with nonlinearities

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Abstract

The investigation of problems that occur nowadays are becoming more and more complicated. Often they can only be solved with the help of numerical methods. The resulting large system of equations to be solved push even modern computer to their limits.

This has led to the development of parallelisation, optimisation and surrogate models. While the former focuses on the more efficient utilization of available resources, the surrogate models are concerned with finding a model with similar physical and mathematical properties that can produce meaningful results with less effort. One possibility to create a surrogate model is proper orthogonal decomposition (POD) [1]. This is a projection based model reduction technique. For this the full system must first be calculated in order to be able to filter out the relevant characteristics, so-called basis functions or modes, via different solutions of the full system. Accordingly, the full system can be projected to a smaller subsystem which can represent these characteristics appropriate. In many applications problems have a small number of characteristics which can describe the problem well.

Problems with more non-linearities requires more modes or the accuracy of the calculation suffers. A larger number of modes decreases the efficiency of the POD method. In order to counteract this problem, an adaptive POD (APOD) [2] method is presented, which looks at the current solution online and selects a set of modes depending on the last timestep, which is good suited for the next timestep. This APOD method is investigated for problems with large deformations, damage and hardening.

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Expected improvement based infill Sampling for full-field multi-fidelity optimization

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Abstract

We are interested in reduction of optimization costs during aircraft engines design process such as Contra-Rotating Open-Rotor (CROR) engines where the quantities of interest derived from a single 2D or 3D solution, physics-based surrogate models (or metamodels). The cost associated to the high-fidelity simulation evaluation makes direct optimization unaffordable for industrial applications. In response to these difficulties we present a multi-fidelity or variable-complexity [1, 2] model to balance its cost and needed accuracy based on the reduced basis approaches with an enrichment (adaptive) strategy on full-field optimization [3, 6]. The proposed multi-fidelity model approximation and the adaptive criteria is validated on a benchmark test function representative of complex features observed in turbomachinery optimizations.

1 The proposed methodology

The presented methodology is aiming to select interesting experiments to evaluate in order to satisfy the constraints at hand, minimize the optimization objectives and/or improve the surrogate model quality, in order to accelerate the descent towards improved configurations in complex industrial optimization problems. The selected approximation models are based on the Non-Intrusive Reduced Basis methodology developed by Maday et al. [4]. Denoting Φ the basis of this linear space, approximating the high-fidelity solution space from M vectors v sampled from the design space \mathcal{D} . Considering a low-fidelity model allows to obtain an approximate solution v_{LF} at a fraction of the cost of its associated high-fidelity resolution v_{HF} , we can build the multi-fidelity approximation $\Phi\Phi^T v_{LF}$, we define the multi-fidelity coefficients as

$$\alpha_{MF}^i = \int_{\Omega} \phi(x)v_{LF}(x, \vartheta^{(i)})d\Omega, \quad i = 1, \dots, M, \quad x \in \Omega. \quad (1)$$

The cost function is built from the approached multi-fidelity model of samples. This approximation is corrected by a stochastic term such that

$$\mathcal{J}(\vartheta) \approx \mathcal{J} \left(\sum \phi(x) \alpha_{MF}(\vartheta) \right) + \Delta(\vartheta), \quad \Delta(\vartheta) \sim \mathcal{GP}(0, \sigma^2) \quad (2)$$

We build a kriging (or Gaussian process \mathcal{GP}) based correction model of the gap between the high- and multi-fidelity cost function. This term is evaluated for the training set then predict the correction to add in order to solve the optimization problem. The expected improvement [5] is applied to the kriging model by solving the maximization problem. The next point is added to improve the model in its maximum gap region.

2 Application

The proposed methodology is illustrated on a bi-level analytical application presented in [6], in order to highlight interesting features in aerodynamical optimization of turbomachinery. The tested benchmark function is defined for a 2D design space such that $\vartheta = (\vartheta_1, \vartheta_2) \subset \mathcal{D} = [4, 6] \times [10, 14]$, $x \in \mathbb{R}^n$ is the discretized position vector of n dimension, such as $n \gg M$, for M samples. The optimization test problem is defined for any continuous function f , with a cost function \mathcal{J} defined as $\mathcal{J}(\vartheta) = \min_{x \in [0,1]} f(x, \vartheta)$. The optimization problem is then

$$\begin{aligned} \min_{\vartheta \in \mathcal{D}} \quad & \mathcal{J}(\vartheta), \\ \text{s.t.} \quad & \arg \min_{x \in [0,1]} f(x, \vartheta) \leq 0.75 \\ & \max_{x \in [0,1]} f(x, \vartheta) \geq 7.5 \end{aligned} \quad (3)$$

The cost function \mathcal{J} is evaluated for the multi-fidelity and the high-fidelity functions f_{MF} and f_{HF} and the problem is solved on figure 1. The figure shows the search of the optimum on the approximate corrected cost function built from the multi-fidelity model (right) and the high-fidelity optimum, cost function and its two constraints (left). The model succeed to reach the optimum for 1 to 5 enrichment high-fidelity points.

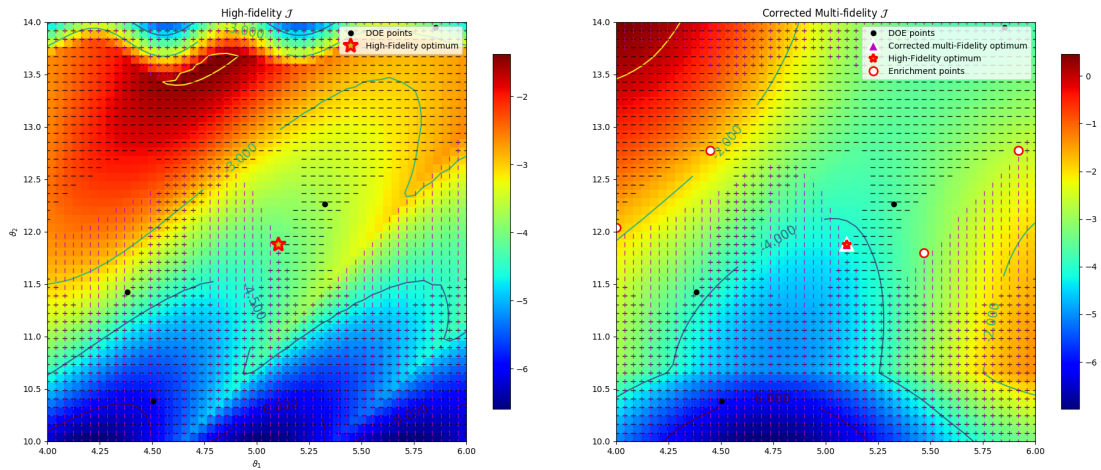


Figure 1: The optimization solutions for high-fidelity and corrected multi-fidelity cost functions

The proposed methodology will be applied to fluid dynamics calculation for the optimization of noise quantities to the bi-level fidelity Contra Rotating Open Rotor (CROR) model. The low-fidelity model is a steady vortex lattice (VLM) method using Sandra. High-fidelity model consists in aero-mechanical-acoustic computational chain with fluid-structure coupling Unsteady Reynolds-Averaged Navier-Stokes (URANS) computations using the ONERA code elsA.

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Reduced Basis Approximation and Goal-Oriented Error Estimation for Time-Dependent Coupled Thermo-Hydro-Mechanical System

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Abstract

In this contribution, a goal-oriented a-posteriori error estimator is developed for a time-dependent coupled thermo-hydro-mechanical (THM) system that assesses the error in specific quantity of interest (QOI). The goal-oriented error estimates are derived based on explicitly-solved weak residual of the primal problem and implicitly-solved adjoint of the dual problem associated with the QOI. The time-dependence of the coupled THM system poses an additional complexity as it involves computation of the auxiliary dual problem evolving backwards in time.

The error estimator is employed to support a greedy-based adaptive procedure that generates an optimal reduced basis by smartly selecting snapshot points over a given parametric training sample. The generated reduced basis is used in a Galerkin projection procedure that results to a model with state-space dimension reduced by several orders of magnitude. An application of this technique is demonstrated through a 3D parametrization problem that investigates the evolution of coupled THM processes in nuclear waste repositories.

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A spot weld simplified model based on reduced order modeling

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Abstract

In an automobile structure, a spot weld (SW) is a pointwise connection element where stress concentrations occur. Therefore, the region will be particularly exposed to fatigue failures, and a SW model for fatigue analysis is necessary. However, the large number of Spot welds (from 4000 to 6000 [1]) requires this SW model to be simplified. Multiple models have been developed to reproduce the SW behavior within a structure [2]. They are mostly phenomenological and with a minimum numerical cost [3, 4, 5]. Therefore, these models are limited [6]. The purpose of this study is thus to propose an accurate model of connector (i.e. with a strong physical meaning, a wide domain of applications), that is based on a methodology adapted to the industrial constraints as a reasonable numerical cost. First, it is necessary to identify the Mechanical Influence Area (MIA) of the SW. In other words, the zone where 3D edge effects spreading around the connector are leading to a no longer valid shell theory. Second, the development of a detailed methodology of construction of the connector model is necessary. From a well chosen basis of loadings, the mechanical response is recreated as a linear combination of spatial plate mode shapes, that are calculated by Singular Value Decomposition. The question of the connection of the calculated deformed modes to the finite element plate model used to model the whole structure, is treated and illustrated. The mechanical response of the prototype is compared to a full 3D finite element models of an elementary welded assembly. An enrichment criteria for the initial basis of solicitations is questioned. The numerical objects necessary for the implementation of such a connector in the software Abaqus V2016[®] are listed.

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Projection-based Model Order Reduction for Model Predictive Control of a Landing Aircraft

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Abstract

A model predictive control (MPC) method seeks to produce a control law that optimizes future behavior of the system over a finite time horizon, by leveraging a model of the dynamics of the system. For rigid systems immersed in a fluid domain, the excessive computational cost associated with solving the fluid/rigid-body dynamics interactions creates a challenge for MPC. Projection-based reduced-order model predictive control (PROMPC) addresses this issue by using a projection-based reduced-order model (PROM) to reduce the cost of the model, though additional work is required to account for the error introduced by the PROM [1]. Typically, a linearized PROM is used, as the controller is expected to maintain the system within the linear regime. This creates a challenge for controlling a descending aircraft due to the unsteady nature of the flow, which is driven by changing atmospheric conditions, whereas standard linearizations are performed about a steady-state. In this work, a linearization about a dynamic rather than static equilibrium is set forth, and a PROM is constructed and exploited to accurately predict the aerodynamic forces acting on an aircraft [2]. Training data is produced by assuming a planar rigid-body motion and solving the linearized equations in the frequency domain. Computational results are demonstrated for the mAEWing2 aircraft using a PROMPC scheme to generate control inputs that are fed to the full-order computational model.

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Some geometric and machine learning methods for model order reduction

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Abstract

Geometric methods have proved to be effective for data mining (simulations, measurements) in the context of nonlinear model order reduction problems in mechanics (e.g. nonlinear methods for dimensionality reduction such as Local Principal Component Analysis, LPCA or Locally Linear Embedding, LLE). In addition, deep learning methods by so-called neural network (NN) architectures proved to be effective for a class of problems involving data classification, detection and compression.

We first present a set of geometric tools for the interpolation of reduced bases obtained by orthogonal decomposition (POD) of simulation results. More precisely, by exploiting the intrinsic geometry of the subspaces engendered by the POD bases, we give the construction of different interpolators on the Grassmann manifold.

Then, we use the architecture of neural networks to extract geometric informations about the data manifold. More precisely, a metric on the manifold can be found from the representations of coordinates learned by the network. Conversely, classical architectures of neural networks can be informed by the geometry of the data to build optimizers that respect the local geometry.

These approaches are illustrated on model order reduction problems in mechanics.

Reduced Model Order Techniques For Data Assimilation In Tailings Dams Monitoring.

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Abstract

The increased demand for mining activity over the last years, has led to an increasing production of enormous volumes of waste, to be stored using tailing dams. The frequent and devastating tailings dam failures have sparked the interest of the mining industry. State-of-the-art techniques developed to improve the monitoring of tailings dams and similar infrastructure, are based on sensor networks that are installed on the structure. The devices collect measurements of pore water pressure and/or other parameters, and report them to a monitoring centre, in real time. To optimize the exploitation of such kind of data for educated decision making, the need to combine the data with a numerical model arose.

The scope of this work is the creation of a model that can receive sensor data of local pressure and/or displacement measurements in real time, and evaluate the full stress and deformation fields in the body of the structure, as well as predict the location of the phreatic surface. The model will be used for optimizing the monitoring system using data assimilation techniques [3], for parameter identification (soil mechanical characteristics, initial conditions) and optimal sensor placement.

An initial FEM model of the groundwater flow through a poroelastic material is presented. It is developed in the FEniCS computing platform, an open-source platform that automates the solution of Partial Differential Equations with FEM [1]. The governing equations describing the two-way time-dependent coupling that features in the Darcian flow of water through a porous medium were developed by Biot [2] in the context of soil consolidation. The model is validated with the aid of benchmark problems and mesh convergence of the method is verified. Subsequently, Reduced Basis techniques are applied to enable the solution of many-query problems and data assimilation. The accuracy and efficiency of the developed low-order solver is examined.

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New numerical tools based on PGD in the context of model identification from full-field measurements

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Abstract

In this work, we introduce a new robust procedure to identify material parameters of mechanical models from full-field measurements. It is based on data information coming from the Digital Image Correlation technique. The procedure aims at defining a suitable numerical processing, in terms of model selection and discretization mesh, with respect to information and noise contained in the data. The nature of the procedure is to minimize a cost functional based on the modified Constitutive Relation Error concept, which is made of modeling and measurement terms. Constructing an admissible stress field, verifying the equilibration equation in a full sense, enables one to obtain estimates on both discretization and modeling errors, which can then be compared with measurement noise in order to drive mesh adaptation and model enrichment. In addition, the procedure is coupled with one of reduced order modeling techniques - Proper Generalized Decomposition - in order to optimize computation costs. The overall approach is implemented on several numerical experiments with linear or nonlinear material behaviors.

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An automated Krylov subspace recycling based model order reduction technique for acoustic BEM systems

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Abstract

Acoustic frequency sweep analyses with the Boundary Element Method (BEM) [1] are often hindered due to the extensive computational cost induced. The combination of a $\mathcal{O}(N^3)$ system solution algorithmic complexity with $\mathcal{O}(N^2)$ storing requirements for a BEM simulation at each frequency line renders the solution for multiple frequency lines a tedious task. Thus, the employment of smart procedures that alleviate the total corresponding computational cost is motivated. In this light, in addition to the existing techniques [2, 3], a new method was recently proposed in [4] for the model order reduction of BEM systems based on the Krylov subspace recycling for sequences of linear systems [5]. The introduced technique constructs a projection basis by collecting the Krylov subspaces of a certain order q , assembled by the BEM systems on a predefined master frequency grid $F_{master} = [f_1, f_2, \dots, f_L]$, and subsequently by following an SVD orthogonalization procedure. Hence, the Krylov subspaces generated at the selected master frequency grid are recycled to construct the solution at the remaining frequencies of the desired spectrum. Nevertheless, due to the implicit frequency dependency of the BEM kernel, the projection of the BEM systems cannot be conducted in a straightforward manner. Instead first, a series expansion of the Green's function kernel is leveraged to transform the BEM system into a series of frequency decoupled matrices that can be projected offline.

Although the technique presented in [4] provides an efficient framework for the model order reduction of BEM systems and is accompanied by an error estimator, the configuration of the employed parameters is based on heuristics, as a constant spacing of the frequency grid and Krylov subspaces order is opted for. In this work, the selection of the parameters in [4] is optimized through an automatic procedure. Specifically, aiming at a predefined error limit ϵ_{lim} , the presented methodology iteratively constructs the projection basis determining automatically the location of the master frequencies F_{master} as well as the adaptive order of the Krylov subspaces q_i generated at each master frequency f_i . The technique is based on the assumption that the highest relative error occurs at the intermediate frequency $f_m = \frac{f_i + f_{i+1}}{2}$ of two consecutive master frequencies f_i, f_{i+1} . This assumption is legitimate considering that Krylov subspace recycling is more efficient for closely related and thus neighbouring linear systems. Additionally, it exploits the influence of the higher frequency Krylov subspaces on the lower frequency BEM systems as

stated in [4] and results in a basis that employs Krylov subspaces of higher order on a more refined frequency grid for higher frequencies.

In order to demonstrate the effect of deploying the proposed automated procedure, the academic example of a unit length cube with a single vibrating face is employed. The mesh of the model consists of 2606 nodes and 5208 constant triangular elements being valid until $f_{max} = 1200\text{Hz}$ for the rule of thumb of 6 elements per wavelength. As illustrated in Figure 1 and Table 1, the automated procedure results in an adaptive configuration for both the spacing of master frequencies and the order of Krylov subspaces. The new reduced order model satisfies the pre-selected error limit $\epsilon_{lim} = 0.01$ for the vast majority of the frequency lines economizing in the same time in the full systems to be assembled and the employed model order. The frequency lines that fail to comply with the targeted error limit coincide with the irregular frequencies of the BEM system.

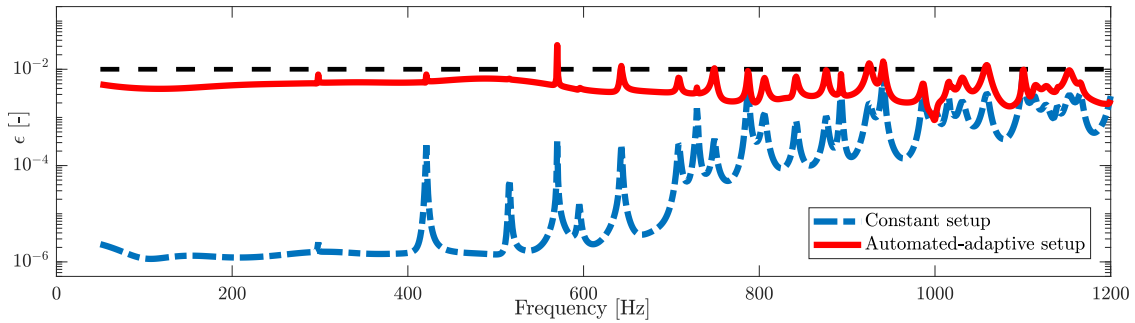


Figure 1: Relative errors yielded from constant and automated-adaptive parameters setup.

Frequencies [Hz]	200	400	600	800	1000	1100	1200	Order
Constant Setup	$F_{master} = [50, 50, \dots, 1150, 1200]$ and $q = 15$							308
Automatic-Adaptive Setup	2	4	5	13	31	25	26	106

Table 1: Order of Krylov subspaces and location of master frequencies for two setups.

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Automatic generation of minimal and reduced models for structured parametric systems

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Abstract

In this work, we discuss a novel model reduction framework for generalized linear parametric systems, with transfer function given by

$$\mathbf{H}(s, \mathbf{p}) = \mathcal{C}(s, \mathbf{p})\mathcal{K}(s, \mathbf{p})^{-1}\mathcal{B}(s, \mathbf{p}).$$

Such systems arise, *e.g.*, in the context of second-order linear systems and delay systems, and they may also have parameter dependencies. Our main goal is to construct reduced-order models for this class of systems. To this aim, we first characterize reachability and observability for such structured parametric systems. Thus, we can construct the numerical minimal realization by projection. Further, we extend the connection between interpolation-based [2] and Loewner approaches [1], known for standard linear systems to the considered class of generalized linear parametric systems, which allows us to construct reduced-order models. Furthermore, special attention is given to computational aspects of the approaches, and we discuss their applicability to large-scale problems. We illustrate the efficiency of the proposed approach with several numerical benchmark examples.

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PGD for 2D elasticity problems, with error bounds and adaptativity

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Abstract

Computational methods have become an essential part of most high end engineering projects, greatly simplifying analysis and design tasks. Reduce order methods, play an important role in this framework, by decreasing the computational effort while maintaining an acceptable level of accuracy. The Proper Generalized Decomposition (PGD) is one of these methods, aiming to solve multidimensional and/or multiparametric problems [1].

This work presents the use of an error estimation method applied to drive adaptivity processes on a PGD solution of a linear elastic 2D problem. It simultaneously seek two complementary PGD approximations, one compatible and one equilibrated, which are used to bound their error (as in [2]) and drive an adaptive process in the space domain.

The strategy to deal with the complementary problem follows the work of [3], approximating Piola-Kirchhoff stresses in a local reference system while enforcing divergence free conditions. The PGD approximations are obtained by implementing the algebraic PGD method from [4], which allows to encapsulate the PGD methodology into a simple and fast algorithm [5].

The results from the adaptivity process show a high level of accuracy, with the estimator capturing the core features of the problem. It detects and refines areas of stress concentration that come from the geometry, and also considers the errors introduced by the variations of the parameters of the problem, such as singularities in the transition between material properties.

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Green's functions of a layered halfspace using the proper generalized decomposition

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Abstract

The fundamental solutions or Green's functions of a layered halfspace correspond to the soil displacements or stresses at receiver location \mathbf{x}_r in direction i due to a concentrated pulse load at source location \mathbf{x}_s in direction j . These functions are required as input in the boundary element method (BEM), which is used to solve dynamic soil-structure interaction problems. Since no closed-form expressions are available for layered halfspaces, the thin layer method (TLM) [2] or direct stiffness method (DSM) [3] is often used to compute Green's functions. Especially for deeply embedded foundations, computing Green's functions for a large number of source-receiver points is computationally demanding. Hence, the fitness of the proper generalized decomposition (PGD) to obtain low-rank approximations of these Green's functions is investigated.

It is assumed that the soil medium is linear elastic and horizontally layered. For the 2D case, wave propagation in the (x, z) -plane is decoupled into in-plane motion (P-SV problem) and out-of-plane motion (SH problem). The TLM is employed to discretize the soil domain up to $z = H$ (figure 1), where it is either resting on bedrock or supported by a homogeneous halfspace of which the stiffness is known analytically [3]. The Green's function is computed in the frequency-wavenumber domain for a number of source-receiver points. This yields a problem in four dimensions: the excitation frequency ω , the wavenumber k_x , the source depth z_s , and the receiver depth z_r . A pro-

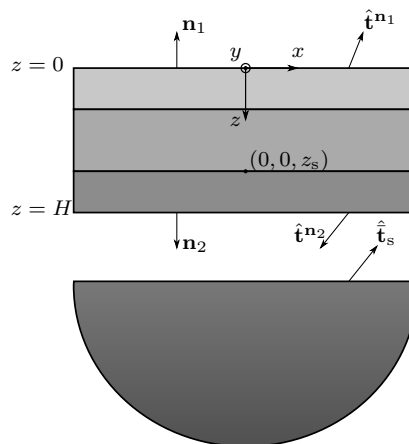


Figure 1: Horizontally layered soil medium.

gressive Galerkin based PGD formulation is used to obtain low-rank approximations of Green’s functions [4, 5].

The methodology is applied to a number of examples. Figure 2 shows the SH Green’s functions for the site in Lincent, Belgium of which the soil properties are given in table 1. The thickness of the layers in the TLM mesh equals 0.1 m. The excitation frequency is sampled between 20 and 40 Hz with a frequency bin of 1 Hz, while 1000 samples are used for the wavenumber domain. The source depth is sampled between 0 and 7 m with steps of 0.2 m. The source is located at $z_s = 2$ m. A low-rank PGD approximation of the Green’s functions with respectively 15 and 30 modes is compared to the reference solution obtained with the DSM. The approximation improves when the number of modes increases. It was found that it is easier to obtain low-rank approximations in a narrow frequency band. For the approximation with 30 modes, the storage requirements are 1600 times lower than the full 4D solution.

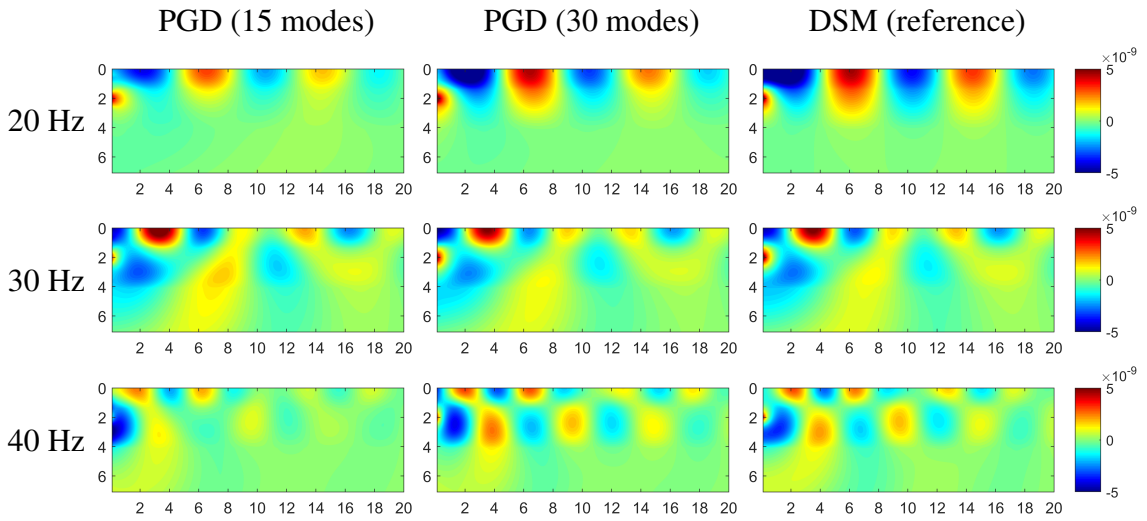


Figure 2: Real part of the SH Green’s functions for the site in Lincent, Belgium.

h [m]	C_s [m/s]	C_p [m/s]	ρ_s [kg/m ³]	β_s [-]	β_p [-]
1.4	128	286	1800	0.044	0.044
2.7	176	286	1800	0.038	0.038
∞	355	1667	1800	0.037	0.037

Table 1: Soil properties of the site in Lincent, Belgium.

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A domain-based adaptive hyper-reduced framework applied to plasticity and crack growth

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Abstract

Accurate analysis of failure in solids often requires the use of computationally expensive modeling frameworks. When modeling ductile failure, constitutive models with complex stress update procedures — *e.g.* plasticity, viscoelasticity, viscoplasticity [1] — are often employed. For fracture, realistic modeling of phenomena such as crack initiation, propagation, merging and branching is commonly performed through regularized methods (*e.g.* Gradient-enhanced Damage, Phase Field, Thick Level Set) that require dense finite element meshes along the crack path [2]. Furthermore, due to the fact that many of these models are based on discrete time integration or staggered analysis schemes, small time steps are usually required to maintain accuracy. These computational bottlenecks effectively render the use of accurate failure models infeasible for many practical applications, especially when used as part of a multiscale framework [3].

Accelerating numerical analysis in solids with Model Order Reduction (MOR) techniques would extend the applicability of the available models. The main idea behind these methods is to find lower-dimensional surrogate solution spaces for the original full-order problems. Projection methods such as the Proper Orthogonal Decomposition (POD) can be used to drastically reduce the number of degrees of freedom of the model, while hyper-reduction methods such as the Empirical Cubature Method (ECM) [4] reduce the effort associated with computing the global internal force vector and stiffness matrix. Surrogate solution spaces are usually computed during an *offline* training phase that includes a number of representative load cases and then used *online* to predict model behavior under general loading conditions.

Although pre-trained models excel in predicting structural response in the linear-elastic regime or under distributed nonlinear behavior, they are less suitable to model localized failure phenomena such as plastic strain localization [5] or damage propagation [6]. This is related to the fact that localized failure leads to large parameter spaces: small changes in load history, direction or location may give rise to significant changes in failure behavior.

In this work, an adaptive reduction framework is presented that precludes the need for an *offline* training phase. The analysis starts with a fully-solved load step and two consecutive reduction procedures are performed: first a POD reduction on the number of degrees of freedom followed by an ECM reduction on the number of integration points. A local/global approach is adopted that allows for the equilibrium of regions responsible for most of the nonlinear behavior to be fully solved while still solving most of the domain

in the reduced space. After each reduced load step, the reduction error is estimated and a fully-solved retraining step is triggered if necessary. The integration error associated with the reduced cubature scheme is controlled by adopting a domain-based reduction approach, with the mesh being adaptively divided into subdomains with different reduced solution spaces.

The framework is demonstrated for both plasticity and quasi-brittle failure through crack growth. For plasticity, a number of strategies are proposed in order to define the fully-solved domain by detecting regions with high energy dissipation rates. The adaptive domain decomposition is performed by grouping finite elements with similar strain response through a k -means clustering algorithm. For crack growth analysis, the framework is applied to the Thick Level Set method [2]. Special strategies are proposed that allow for the reduced model to cope with a dynamic number of integration points and an adaptive nodal enrichment scheme that leads to a change in the size of the full-order problem as cracks propagate. The level set field used to locate the damaged zone is exploited in order to define the fully-solved domain and the different hyper-reduced subdomains. The performance of the framework is investigated through a number of numerical examples.

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A new time multi-scale LATIN-PGD for the treatment of complex seismic excitations

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Abstract

The increasing complexity of the numerical models used to predict the (often strongly nonlinear) seismic behaviour of reinforced concrete structures impose large computation time for solving the partial differential equations of the reference problem. Furthermore, margin assessment, stochastic analysis or model updating generally impose to make this computational effort, not only for the simulation of a single model, but of a family of models. In addition to the uncertainties associated to material parameters, a major lack of knowledge also concerns the excitation itself. The seismic risk is usually defined specifying parameters (magnitude, fault mechanism, distance to site, shear wave velocity, etc.) that describe a likely fault scenario. The mapping between those parameters and a probable seismic signal is far from being bijective ; from a unique parameter set, an infinity of likely ground motions with common statistical properties can be generated (see e.g. [1]). The approach that will be followed must incorporate the fact that the numerical models will be confronted to numerous signals with common properties.

To reduce the computation cost, dedicated strategies referred as "model reduction techniques", are usually considered. Among the most effective, the LATIN/PGD method has proved to drastically decrease the computational cost for the resolution of nonlinear problems. It is also particularly efficient for treating parametric problems as for materials under quasi-static loading and with typical elastic-visco-plastic behaviour [2] or more recently [3] on reinforced concrete modelling. For evolutionary problems, the LATIN/PGD approach consists in a sequence of *(i) local steps* where constitutive relations are solved locally on all the integration points in space and time, and *(ii) global steps* where the equilibrium on the whole space-time domain is imposed. The most numerically expensive task is by far the re-imposition of global equilibrium at step *(ii)* ; however, this step is linear and its solution can be approached using a low rank approximation, known as the *Proper Generalized Decomposition* (PGD), which greatly improves the numerical performance of the approach.

A first step at this work will be the adaptation of the LATIN/PGD approach to low dynamics, typically considered for earthquake engineering problems. A second step concerns the complexity of the seismic input itself. Previous works on fatigue for damageable visco-plastic structures have proposed an adaptation of the LATIN/PGD method for a drastic reduction of the computational burden for the case where the excitation is mono periodic and composed of thousands cycles [4], and more recently in a refreshed version in [5]. To benefit from this last improvement, an original decomposition of the seismic signal (input excitation), and the residual time function that arise at the linear step once the nonlinear constitutive relation is integrated was proposed in [6], such that the work presented in [4] can be extended for a multi-frequency case, leading to a resolution in time on a coarse macro scale discretization rather than the original and finest discretization of the problem.

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Temporal dimension reduction via parametric state-time formulation

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Abstract

The majority of model order reduction (MOR) methods focusses on the dimension reduction of high-order model equations, which typically arise from spatially discretized partial differential equations (PDE). Besides the explicit and typically significant influence of the MOR methods on the order of these types of model equations, the temporal dimension in transient analyses is only implicitly and not always significantly influenced.

Another category of models are low-order lumped parameter model equations, which are among others used to describe system-level topologies of machine designs. Although the state order is low, the computation of the optimal parameters based on an objective function which encompass the evaluation of a transient simulation is computationally expensive or even intractable.

Therefore, a novel state-time parametric reduction method based on the work of Choi and Carlberg [1] is developed, which reduces the temporal dimension of the transient analysis, rather than the state dimension of the model equations. The proposed method reduces the time-discretized algebraic system

$$\mathbf{D}\chi = \mathbf{s}(\chi, \mathbf{x}_0, \mathbf{p}), \quad (1)$$

with the state-time unknown $\chi = \{\mathbf{x}^T[1], \dots, \mathbf{x}^T[n_t]\}^T \in \mathbb{R}^{(n_s \cdot n_t)}$, the initial condition $\mathbf{x}_0 \in \mathbb{R}^{n_s}$ and the state difference $\mathbf{D} \in \mathbb{R}^{n_s \cdot n_t \times n_s \cdot n_t}$ and system specific $\mathbf{s} \in \mathbb{R}^{n_s \cdot n_t}$ contribution of the time discretization [2]. Equation (1) is reduced via a Galerkin projection with a basis computed via a higher-order singular value decomposition of training data. The nonlinear term $\mathbf{s}(\chi, \mathbf{x}_0, \mathbf{p})$ is furthermore hyper-reduced via an adapted DEIM approach. As a result, the evaluation of the reduced algebraic system or state-time ROM (STROM) requires only the (parallel) evaluation of the state propagation at certain time steps.

The key research result is the significant reduction of the temporal dimension while maintaining a high accuracy. The reduction method is applied on a cardanic drivetrain, shown in figure 1, containing among others non-linear kinematic relations. Figure 2 shows the accuracy of the method in function of the order of the state-time reduced model and in function of the number of time step evaluations.

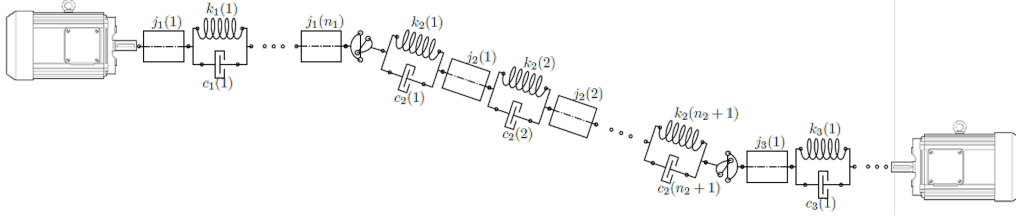


Figure 1: Cardanic drivetrain with non-linear induction motor model and non-linear kinematic cardan joints. (order model $n_s = 14$; time step $h = 0.1ms$)

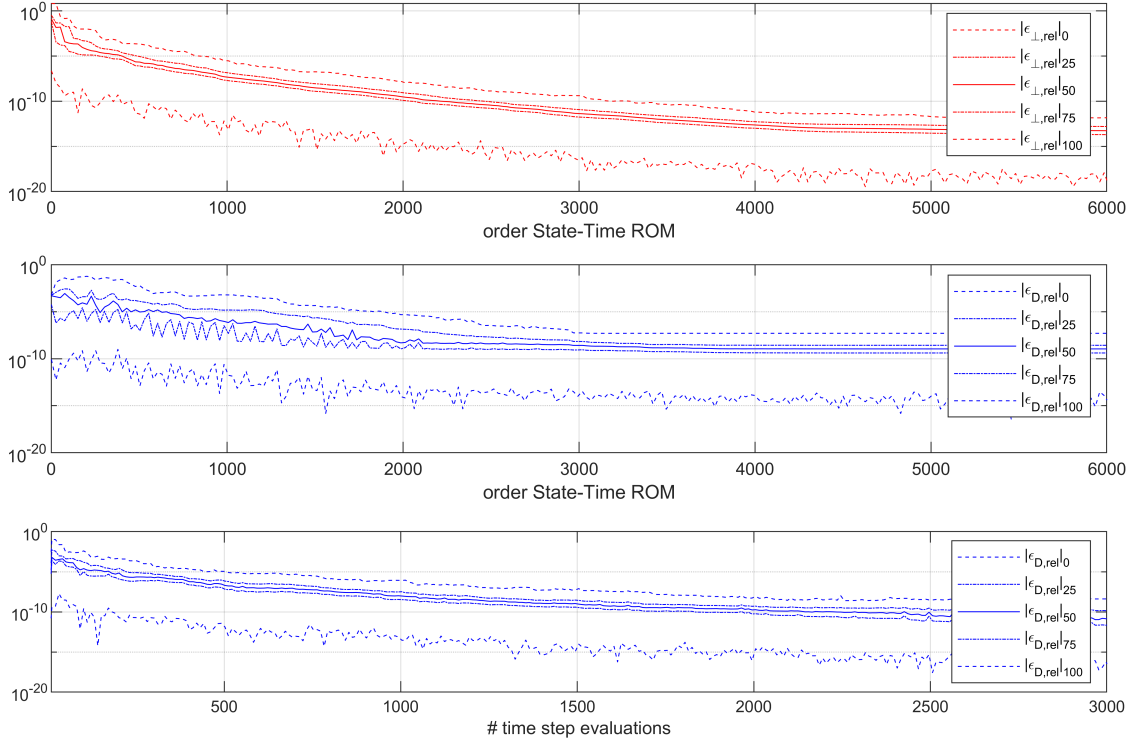


Figure 2: Error analysis of temporal dimension reduction for a simulation of 1s ($n_t = 10,000$) of the model shown in figure 1.

subplot (1-2): Out-of-manifold relative error and hyper-reduction error in function of STROM order. (number of DEIM time step evaluations is 1400)

subplot (3): hyper-reduction error in function of number of DEIM time step evaluations. (order STROM is 4000)

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On a non-intrusive version of the LaTIn-PGD method for non-linear time-dependent problems

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Abstract

Reduced-order modeling methods occupy a prominent place in numerical simulation : strategies such as Proper Orthogonal Decomposition (POD), Reduced Basis (RB) or Proper Generalized Decomposition (PGD) [1] have largely proven their value in drastically reducing CPU time [4, 5]. However, the development of these reduced-order modeling approaches in the industrial world is currently hampered by the difficulty of introducing these methods into finite element codes. Indeed, the desire to be able to manipulate reduced-order models in a single software platform is often opposed by the strong intrusiveness of the associated algorithms. This work introduces a non-intrusive reformulation of the LaTIn-PGD method [3], in the form of generalized quantities, which is intended to be directly integrated in finite element analysis softwares. Such an approach should significantly reduce the computation time for all complex time-dependent non-linear problems that these codes can handle, insofar as the non-linear LaTIn-PGD solver [2] will allow to use all non-linearities, element types, tools, ... already included in finite element softwares. As part of this work, this approach is carried out in the non-linear finite element analysis software Samcef developed by SIEMENS. In the first place, the feasibility of the method is highlighted by means of a prototype algorithm for an elasto-visco-plastic material.

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FV-based POD-Galerkin reduced order model for boundary control of natural convection flows

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Abstract

A Reduced Order Model (ROM) for unsteady natural convection is developed for which the Full Order Model is based on the finite volume approximation. The driving force for natural convection is buoyancy, a result of local density differences of the fluid induced by local temperature differences. The system of equations to be solved numerically for this phenomena is highly non-linear due to a two-way coupling between the non-linear momentum and energy equations. The Boussinesq approximation is applied to simplify the problem by neglecting the effect of local density differences, except for the influence of the gravitational body force on the flow.

The reduced basis for the ROM is constructed using a POD approach, which is combined with the Galerkin projection where the governing equations are projected onto the reduced basis and the difference between the reduced- and the full order solution is minimized.

The ROM is tested on a 2D square enclosed cavity with differentially heated walls opposite to each other. The wall temperatures are parametrized. However, non-homogeneous BCs are, in general, not satisfied in a POD-based ROM. Moreover, the BCs are not explicitly present in the ROM and consequently they cannot be controlled directly. Therefore, a control function method is applied and for this test case, the control functions are obtained by solving a Laplacian function for temperature.

The results show that the ROMs are stable and can be used for sensitivity analysis by controlling the non-homogeneous Dirichlet boundary conditions. Finally, a speedup of about 20 times on a single processor is obtained.

Multitemporal space-time integration for dissipative solids under cyclic loading

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Abstract

The calculation of the non-linear response of solids and structures under cyclic loading conditions by means of conventional incremental procedures often leads to intractable computational costs. Several methods have been proposed in the literature to alleviate these issues, aiming to reduce computational time and storage requirements.

Non-intrusive procedures are typically based on cycle jumping techniques [2], where step-by-step calculations are performed for a reduced number of cycles, with an adaptive extrapolation of internal variables in between. Other techniques are based on model order reduction (MOR). A well-established example is the Large Time Increment Method (LATIN) [1], where the full time history of the displacements and internal variables is computed in each iteration. A crucial aspect of the LATIN solver is the decomposition of space-time dependence in the global equilibrium equations, where an a priori MOR technique is employed. This technique can be viewed as a particular case of the Proper Generalized Decomposition (PGD) [5]. Without relying on the full LATIN scheme, PGD has also been used as a space-time integrator for problems in elastoplasticity [3, 4].

In this study, we propose a multitemporal space-time integration scheme based on PGD, intended to describe failure in solids undergoing cyclic plasticity and fatigue. The proposed methodology extends previous developments in space-time PGD and can be applied to a general class of models with internal variables, suggesting possibilities for significant computational savings. The capabilities of the PGD-based procedure are evidenced by benchmark problems that present complex nonlinear cyclic responses. An example is shown in figure 1, corresponding to a perforated plate under force control that exhibits cyclic plasticity. The spatial distribution of the displacements results from localized plastic strains that are governed by an internal length scale in the context of gradient plasticity. On the other hand, the temporal evolution indicates a cyclic hardening response, as well as ratcheting and shakedown phenomena.

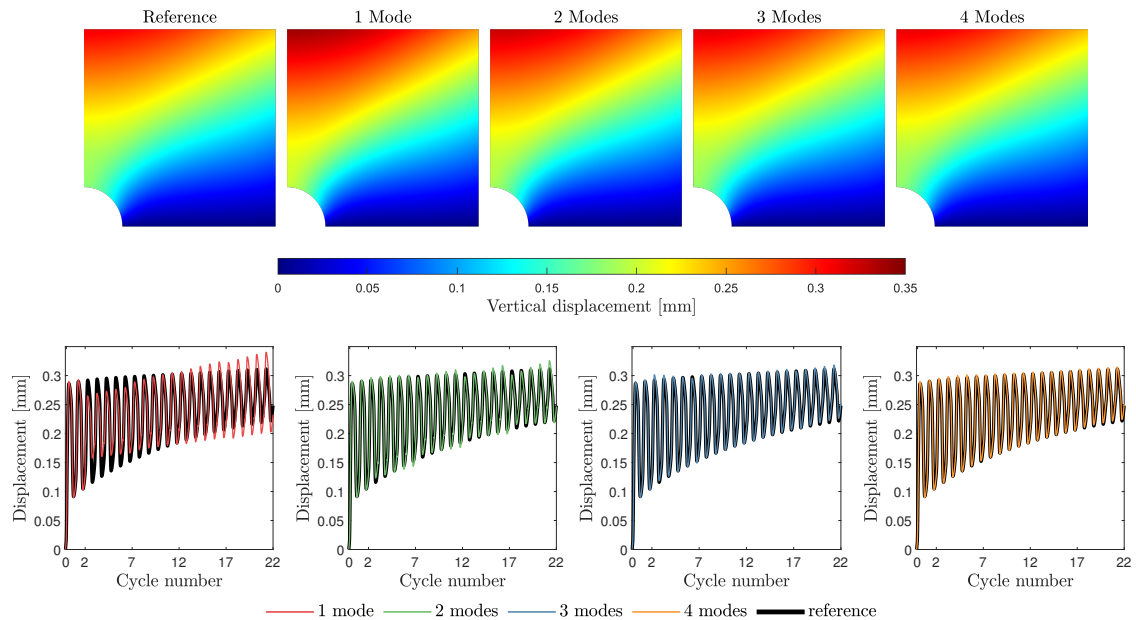


Figure 1: Reference incremental solution and PGD-based solution with different mode numbers for a perforated plate under vertical cyclic loading: spatial distribution of the vertical displacements (top) and temporal evolution at the top-left corner of the plate (bottom).

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Swirling flow analysis in piping system using proper orthogonal decomposition method and Galerkin Projection

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Abstract

In industrial plants such as power generation, petrochemicals and water, the demand for predicting the life of the plant by visualizing the deterioration status of the plant such as pipe thinning and utilizing it for operation and maintenance planning is expanding. To visualize the pipe wall thinning phenomenon, it is effective to calculate the flow state such as the separated flow and the swirling flow in the piping system by three-dimensional fluid analysis and evaluate the pipe wall thinning rate from the calculated flow state. As the scale of three-dimensional fluid analysis in piping systems is large, it is required to shorten the computation time.

To reduce the computation time, an approach that derives a reduced order model (ROM) without losing flow field information for three-dimensional fluid analysis results, and calculates a flow field in a short time using ROM has been proposed. In this approach, low-dimensional basis space is extracted from the result of large-scale high-dimensional three-dimensional fluid analysis by proper orthogonal decomposition (POD), and based on Galerkin Projection, the basic equation of fluid is projected on the basis space to construct ROM. In the previous work, the results of applying POD to cavity flow and straight pipe swirl flow has been reported [1]. In this paper, we report the result of applying POD and Galerkin Projection to swirling flow analysis in piping system.

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Reduced Order Models for incompressible turbulent Navier-Stokes flows in a finite volume environment

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Abstract

This work presents Reduced Order models designed to deal with both turbulent flows and pressure stabilization related to incompressibility.

In order to have an accurate turbulent model, we merge projection-based methods and data-driven techniques. The model utilizes classical Galerkin-projection method for the build up of the Reduced Order dynamical system and for solving for the reduced degrees of freedom for velocity and pressure. Instead a data-driven technique for the approximation of the solution manifold of the eddy viscosity field is used [1].

As regards pressure stabilisation, it is a matter that occurs both at the Full Order and at the Reduced Order levels, thus several different strategies have been developed in order to overtake the obstacle.

In our work we look at obtaining a new path towards the right solution. In particular the idea is to follow the common segregated algorithms, which are quite widespread in almost all finite volume solvers, also at the reduced level so that it is possible to have both a stable pressure recovery and a coherent reduced procedure at the same time [2].

In this session we present the details about this procedure and we show some results obtained by its application. In particular also some comparisons between this architecture and some different other stabilisation techniques [3] will be exposed.

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